ORDERING LOADING STATIONS ALONG A DELIVERY CONVEYOR

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ORDERING LOADING STATIONS ALONG

A DELIVERY CONVEYOR

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SUMMARY

The problem of arranging work stations in series along a continuous conveyor has been investigated in this research. An analytic model describing the expected delay for each station was derived. Also, a simulation model was developed to find the delay for each station. Also, both of these models used station production rates which are Poisson. Furthermore, the stations were not restricted to having identical production rates and box lengths. Both of these models were used in an investigation of a number of station arrangement strategies. Using Friedman's multi-sample test, the rank order of the strategies in terms of lowest arrangement delay was statistically tested. The general findings of the investigation can be summarized as follows:

1. For constant box length, the strategy of placing the slowest station first was found to outrank the other strategies used;

2. For a constant production rate, the strategy of placing the station with the shortest length first was found to outrank the other strategies used;

3. For a situation in which the production rates and box lengths vary from station to station, the strategy of placing the station with the smallest product of rate and length first was found to outrank the other strategies used.

The arrangement rules seem to work best when the conveyor utilization is low. However, their performance as compared to other strategies may not be desirable when the conveyor utilization is high or roughly over fifty percent.

CHAPTER I

INTRODUCTION

The problem of material handling in industry has long been recognized as important. This is due primarily to the large investments of money needed in creating and in maintaining a material handling system. One element in material handling is the transportation of objects from one area to another area of a facility. Many devices are now being used in industry to perform this activity. Cranes, forklift trucks, conveyors, and other similar systems or equipment may provide adequate service, depending on the nature of the items to be transported. If a continual flow from one area to another area is desired, then conveyor systems could be appropriate. There are a number of types of conveyors which could be considered. The following nonexhaustive list briefly describes several types:

1. Roller conveyors, consisting of a series of rollers over which objects can be moved;

2. Hook conveyors, consisting of a moving endless band on which objects can be placed on equally spaced hooks;

3. Belt conveyors, consisting of a moving endless band on which objects can be placed on its surface;

4. Bucket conveyors, consisting of a moving endless

band on which objects can be placed in equally spaced buckets. The proper choice of conveyor type depends upon the total system's costs and its inherent compatability with the material that is to be transported.

When human elements are involved in conveyor systems, a stochastic element is introduced. This is due to the fact that if this human element is not mechanically paced, there will be some variability in its completion. Also, the time needed to manually handle each item will vary. Thus, the man-machine interaction in manually loading a conveyor is a stochastic process.

This research is concerned with the interactions of these human elements with the mechanical conveyor elements.

Problem Statement

The conveyor system to be studied consists of a number of work stations and a continuous delivery conveyor. A work station is that physical area along the conveyor in which a number of operations are performed on one item at a time. Once these tasks are completed, the item is placed onto the delivery conveyor. The delivery conveyor removes the items from the station permanently, such that once removed, an item can never return to that station by the way of the conveyor.

The objective will be to find rules specifying how to arrange stations along a continuous delivery conveyor in order to reduce operational costs in delays. An analytical model will be developed for predicting delays occurring at each station. A simulation model will then be used as a means of checking the analytical model's performance. By using these two models, it will be possible to study various arrangements of different types of stations along a conveyor. From the results obtained from this study, the preferred rules for ordering work stations will be given.

Previous Research

Modern conveyor design theory is usually stated to have been founded by the work of Kwo (9, 10). Kwo's major contribution to this theory was that he made the step beyond the mere specification of the mechanical component parts to an approach that observed the entire system. This entire system consists of the conveyor itself, the loading area, the unloading area, and the interaction of flows between these two areas. This theory was extended and formalized by Mayer (11) and by Morris (12). Recently, the work of Kwo has been challenged by Muth (13, 14, 15), who has presented a less restrictive approach. Extensions of this general theory to more specific problems seen to have fallen into two definable areas. One area is the research related to the queuing analysis of the ordered entry conveyor system. The other area is the research directed to an analysis of work stations along the conveyor and to the establishment of appropriate work strategies. Before discussing these areas, the developments achieved by Kwo will be discussed.

Kwo's Work

The theory of conveyors prior to Kwo's developments was primarily restricted to mechanical specifications. However, two problems developed with operating conveyors. One problem was that at a given loading station it may be impossible to place an item on the conveyor, due to having something already on the conveyor at that point. The other problem was that at a given unloading station it may be impossible to remove an item from the conveyor, due to having no load on the conveyor at that point. The corrective actions normally taken at that time were to increase the speed of the conveyor, to reserve floor space, to convert the system into two systems, or to replace the conveyor by some other type of system. Kwo (9) suggested that the real problem was one of recognizing the total system. The conveyor is not a separate system by itself but a part of a larger system which also includes the loading area and the unloading area. The conveyor is a means of transporting items and of storage. If the rate of loading the conveyor is always equal to the rate of unloading, the total number of items on the conveyor will remain constant. Such conveyors are commonly referred to as delivery conveyors. If these two rates are allowed to differ and vary with time, the total number of items on the conveyor will also vary. At times there will be an increase and at others a decrease in this total number. Thus, the conveyor will perform as a delivery conveyor and also serve a storage function. The conveyors which perform both of these functions are generally referred to as storage-delivery conveyors.

The storage-delivery conveyor system was the subject of Kwo's analysis. There were a number of other assumptions made about this system.

1. The conveyor was a hook conveyor which has receptacles for placing items at fixed intervals, as opposed to a continuous belt.

2. Once the speed was determined for the conveyor, it would remain constant.

3. The rates of loading from a single loading station and of unloading from a single unloading station would be constant.

4. The amount loaded onto the conveyor would equal the amount unloaded at some time.

5. The conveyor itself moved in a loop in which items returning to a given point were permitted.

In his study of such a system, Kwo (9) derived three fundamental principles for conveyor operation and specified an approach for finding a solution for conveyor design and operation problems. This conveyor system, with minor alterations, has been the subject of most of the research in modern conveyor design theory.

Kwo's three principles are the speed rule, the capacity constraint and the uniformity principle.

1. The speed rule defines the feasible region of speeds. This region exists between the lower limit which is defined by the loading or by the unloading rates and the upper limit which is defined by the maximum speed at which the conveyor can mechanically operate or by the maximum speed that allows effective handling of items on the conveyor.

2. The capacity constraint specifies that the conveyor has the ability to adequately store items.

3. The uniformity principle states that the conveyor must be loaded uniformly throughout its entire length.

By using these three principles, Kwo (9, 10) was able to specify a simulation procedure to find solutions for the conveyor operation problem and for the conveyor design problem. In the conveyor operation problem, the only controllable variables are the loading-unloading schedules and the speed of the conveyor. This problem generally reduces to finding the conveyor speed and the prestored quantity for a given schedule. For the design problem, the situation is more complex, since all of the conveyor parameters are allowed to be determined by the designer.

Mayer's and Morris's Extensions

Mayer (11) extended Kwo's work by presenting a methodology, based upon probability theory, for obtaining operational characteristics of a delivery system. This system is described as being composed of N identical stations along a hook conveyor. The number of times that an operator cannot place an item on the conveyor and must place it on the floor is observed. Since the operator is allowed only one attempt in loading an item onto a single hook before placing it on the floor, a series of Bernoulli trials are made on a single hook as it passes the loading stations. By obtaining probabilities associated with this procedure, it is then possible to describe the performance of such a system. Ideal performance is stated as being that situation at any loading station in which all items are accepted by the conveyor without delay or possible rehandling. Thus, the number of items not accepted by the conveyor will provide an index of the system's performance. This index is referred by Mayer as the Measure-of-Demerit.

Morris (12) extends Kwo's work by presenting a detailed description of various aspects of conveyor design theory. The aspects relevant to this research are the discussion of loading and unloading systems, delivery systems, and delays. In his discussion of loading and unloading systems and of delivery systems he presents a conveyor system which is very similar to Mayer's delivery system. The assumption of Berneulli trials is used. The failure to load during a single attempt at the loading station and the failure to be able to remove an item during a single attempt at the unloading stations are referred to by Morris as being possible interferences on the conveyor. As earlier suggested, in process inventories can be maintained to handle this problem. However, Morris suggests that what is desired is to design a conveyor that reduces the possibility of this interference and thus to reduce the necessity for large in-process inventories.

In developing a procedure to describe the system, Morris defines the mean time between loading attempts and the mean loading attempt rate. If the time between loading attempts, t, is a random variable with a density function f(t), the mean time between loading attempts is

$$\int_0^\infty t\ f(t)dt = \bar{t}$$

provided that f(t) is independent of the speed of the conveyor and of the previous attempts to load the conveyor. The mean loading attempt rate is

$$\frac{1}{\bar{t}} = \lambda$$

which is used for every station in the loading "area".

Upon completing his derivation of the probabilities associated with the expected number of loaded carriers, Morris observed that this proportion increases as the conveyor passes loading stations in the loading area. Thus, the loading stations "downstream" from the first loading station will increase in experiencing interferences. Morris states that to reduce this possibility one should increase the speed of the conveyor as it passes these identical stations. However, due to the speed constraint presented by Kwo it is known that this cannot proceed indefinitely.

In this discussion of delayed loading and unloading, Morris changes one of the assumptions made by Mayer. At the loading station if an attempt has failed to load an item onto the conveyor, the operator now must wait until it is possible to do so. In other words, the possibility of placing items onto the floor no longer exists. Since the assumption of discrete entry points on the conveyor is still maintained and by using the geometric distribution, Morris derives several expressions to describe the system. As in the case of the delivery system, it was found that by increasing the speed of the conveyor, the average delay for a station is reduced, and its actual loading rate onto the conveyor will be increased.

Muth's Approach

The work of Morris (12) and Mayer (11) is based on the underlying principles made by Kwo (9, 10). However, it has been stated by Muth (13, 14, 15) that the uniformity principle leads to a subset of the actual feasible candidate set for conveyor operation. Muth (13) states that the design problem or the operation problem could be solved if conveyor parameters compatible with the operating requirements are found. Furthermore, the input and output flows of the conveyor system can be described by a difference equation, and that Kwo's two station system can be converted into an equivalent one station system. Once this is done, Muth proposes a methodology for finding a compatible solution for discrete and continuous cases.

The Ordered Entry Conveyor System

After the work of Morris the research into conveyor theory seems to have diverged into two areas. One of these areas was the research related to the queuing analysis of the ordered entry conveyor system in which items are assigned to the first station along a conveyor in which its placement is feasible. Disney (5, 6) was the first to research this area formally. He decomposed the queuing system into an "overflow process" and derived an m-channel queuing system. This was based upon the assumptions of identical stations and of exponential service times and interarrival times. Pritsker (17) has derived a general m-channel case for such a system. Phillips (16) has derived an m-channel queuing system where the distribution of interarrival times are gamma and exponential for the service times. El Sayed, Proctor and Elayat (7) developed a two channel queuing system that used Poisson arrivals. Also they allowed this conveyor system to use two types of loads.

Work Station Analysis

The other area which comes after Morris's work was the research directed to the analysis of individual and of combined work stations along a conveyor and to the establishment of work strategies that would improve performance characteristics of the conveyor system. It appears that the research presented in this area falls into three branches. In the first branch a general description of individual work stations along a conveyor is given. In the second branch the development of simplified work policies for a single station is presented. In the third branch an analysis of a number of stations in series along a conveyor is presented.

The general description of the individual work station is given by Reis, Dunlap and Schneider (19) and by Reis and Hatcher (20). Reis, Dunalp and Schneider (19) state that there exists for a conveyor system a need to formulate policies regarding the conveyor capacity, the stations and the storage, or banking, allowed at each station. Also, there exists a need to know other aspects of the individual station. In describing these other aspects both articles here used the same terminology. For a loading station there exist three major ranges.

1. The loading range is that distance along the conveyor that an operator is instructed to examine before placing a finished item into the bank, when it is discovered that it will be impossible to place it on the conveyor within this range.

2. The visibility range is that actual distance along the conveyor that the operator can observe from a given station.

3. The bank removal range is similar to the loading

range and is used to remove an item from the bank, provided that it is possible to load it onto the conveyor.

Also, it was pointed out by these two articles that the time required for each station to prepare a unit for loading is the actual productive effort of the work station. Any delay caused by interference with placing a unit onto the conveyor will cause the actual production to be less than the potential production. Reis, Dunlap and Schneider (19) and Reis and Hatcher (20) have examined these delays by probabilistic means. In both articles the equation of actual production for a single station is given. If D is taken as the delay per unit at a station, M is taken as the potential non-delayed production rate at that station, and P is the actual production rate at that station then

$$P = \frac{M}{1 + DM} \quad . \qquad 1\text{-}1$$

The development of simplified station work policies is presented in the work by Crisp (3), Reis, Brennan and Crisp (18) and Beightler and Crisp (1). The main result of the research was the development of two work policies which involved banking. Crisp (3) and Beightler and Crisp (1) describe a method of station operation which is referred as being the Sequential Range Policy. For a loading station, this policy required the following steps to be performed.

1. The operator completes some productive activity on

a new item and immediately stores it in a bank.

2. The operator then observes the loading range to determine if an item from the bank can be loaded onto the conveyor.

3. If it is possible to load an item, the operator will do so and will observe the loading range now starting again from the point at which the item was placed onto the conveyor. In such a manner the operator will continue to load the conveyor until the loading range is full or until the bank is empty. Once either possibility occurs, the operator will return to work on a new item.

4. If it was found that the loading range was initially full the operator would check the bank. If it is not full, the operator will return to work on a new item. However, if the bank is full, the operator will wait until the items in the loading range have passed. Once this has occurred the process will return to step 2.

The method of station operation described by Reis, Brennan and Crisp (18) is the Fixed Range Policy. For a loading station this policy is exactly the same as the Sequential Range Policy except for the policy of altering the loading range in step 3. The Fixed Range Policy prescribes that once a range is specified by the second step it will not be changed by any subsequent loading of the conveyor in the third step. Beightler and Crisp (1) have compared both of these policies and have found that the Sequential Range Policy is superior in minimizing delay at a given station. Both of these two policies have been used in observing a series of stations.

Several analyses of work stations in series are given by the work of Crisp, Skeith and Barnes (4), by Brennan (2) and by Gregory and Litton (8). Crisp, Skeith and Barnes (4) discovered that the Bernoulli assumption used by the single station Sequential Range Policy and by the Fixed Range Policy, would become invalid when it was used with a number of stations in series. Also, Brennan (2) found that for a system of N identical loading stations, using either constant or log-normal production rates, not hold. Gregory and Litton (8) have studied a series of unloading stations along a hook conveyor. They found that, by arranging these unloading stations in order of descending service rates along the conveyor, the number of items missed by the stations would be minimized.

Upon surveying the research concerning work stations in series, there appears to be an area unexplored. Crisp, Skeith and Barnes (4) and Brennan (2) have studied the case in which the work stations are in series along a hook conveyor and are identical, i.e., having the same service rate. Gregory and Litton (8) have found a solution for ordering stations along a hook conveyor with different productive rates. However, the scope of their research was limited to the unloading area. What has not been touched in the issue of having loading stations in series along a continuous conveyor in which the productive rate at each station may be different and the length of what is being placed on the conveyor may also be different. This research will observe such a system and, unlike Gregory and Litton (8), will use the amount of expected delays at each station as the measure of effectiveness for the system. The system that will be observed is a delivery conveyor system. It was previously mentioned that both Mayer (11) and Morris (12) have analyzed the delivery conveyor system. However, neither took into account the possibility of having non-identical stations. In addition, the study of a continuous spaced conveyor has not been explicitly addressed in previous research.

CHAPTER II

MODEL DEVELOPMENT

In this chapter an analytic model is developed to describe the expected delays and reduced production rates for a number of stations along a conveyor. The conveyor system which is considered is a representation of a belt or live roller, delivery conveyor that has a number of sequentially positioned loading stations along it. This system is illustrated in Figure 2-1.

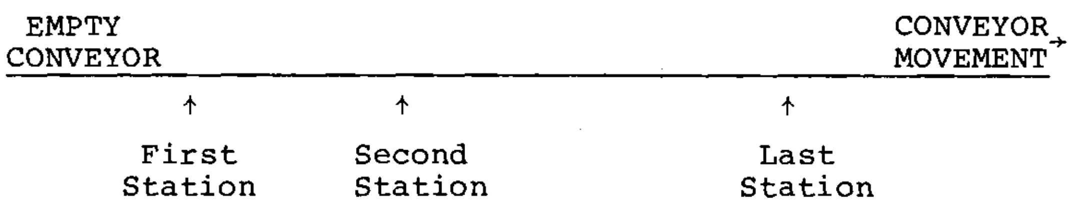

Figure 2-1. A Delivery Conveyor

There are a number of assumptions which have been made about the conveyor itself. They are:

1. It is a delivery conveyor; no items are on the conveyor upstream from the first loading station.

2. It operates at a constant speed over its entire length.

3. Items can be placed at any point along its entire length, as opposed to hook conveyors which can only carry items

at discrete points.

4. It is not wide enough to place items side by side.

Also, there are a number of general statements that describe the work stations. These include:

1. The stations are not required to be identical in regard to production rate or product.

2. The station's productive rate is described by the Poisson distribution, and its output, referred to as a box, is characterized by the length it occupies on the conveyor.

3. The work station's physical dimensions are defined only by a non-overlapping entry range located along the conveyor. An entry range is the distance along the conveyor in which the operator of a station is permitted to place a box. In order to simplify the modeling needed this range is set equal to twice the station's box length. The operator places a box as close to the middle of this range as is possible without incurring additional delays.

4. If, for any reason, the operator cannot load the conveyor without interferring with other previously loaded material on the conveyor, the operator will be required to wait for the first space of sufficient length to arrive within the entry range.

The Two Station Model

The formulation of the two station case is presented in this section and will be used subsequently in developing the three station model and the N station model. Introductory material will be given first to further define the conveyor system. The method for finding the expected delay per cycle for the second station will then be presented. Also, a procedure for determining the adjusted production rate for the second station will be given. Once this has been completed, a methodology for determining the expected box departure rates from the second station will be shown.

Three conditions of the entry range can be observed initially by an operator who attempts to load a box after completing some production activity. At the midpoint of the loading range (M) there may be:

a) a space between two boxes, which is large enough for the box to be loaded, i.e., $s \geq L_i$

b) a space between two boxes that is not long enough, $s < L_i$,

c) a box.

The space that occurs in condition (a) will be referred to as L-space. The space that occurs in condition (b) will be referred to as a S-space. After the conveyor is observed initially, the operator will have to wait for the first L-space to arrive if conditions (b) or (c) were found. This is shown in Figure 2-2 for the second station. The "distance of delay" refers to the length of conveyor which must pass the point M before a delayed box at station 2 can enter.

To find the time for such delays, the "distance of delay" needs to be divided by the velocity of the conveyor, v.

The tree diagram in Figure 2-3 describes more completely what has been shown in Figure 2-2. The following defines the notations in Figure 2-3.

$P(B)$ = the probability of having a box at M

$P(S)$ = the probability of having a space at M

$P(S<L_2)$ = the probability that the space found initially is a S-space

$P(S \geq L_2)$ = the probability that the space found initially is L-space

b = the portion of a box initially found at the midpoint that has not yet passed M

d = the portion of a S-space initially found at the midpoint that has not yet passed M

$P(s<L_2)$ = the probability that the distance between two boxes is less than L_2

$P(s \geq L_2)$ = the probability that the distance between two boxes is greater than L_2

Figure 2-3 is used in deriving the model for the second station. The expected delay per cycle can be found by obtaining expressions for each parameter above and then finding the expected delay for each.

It can be seen from the tree diagram that should the operator find a box initially, P(B), a portion of that box, b, must first go by. Once this has happened there may be a space between that box and the next box coming on the convey-

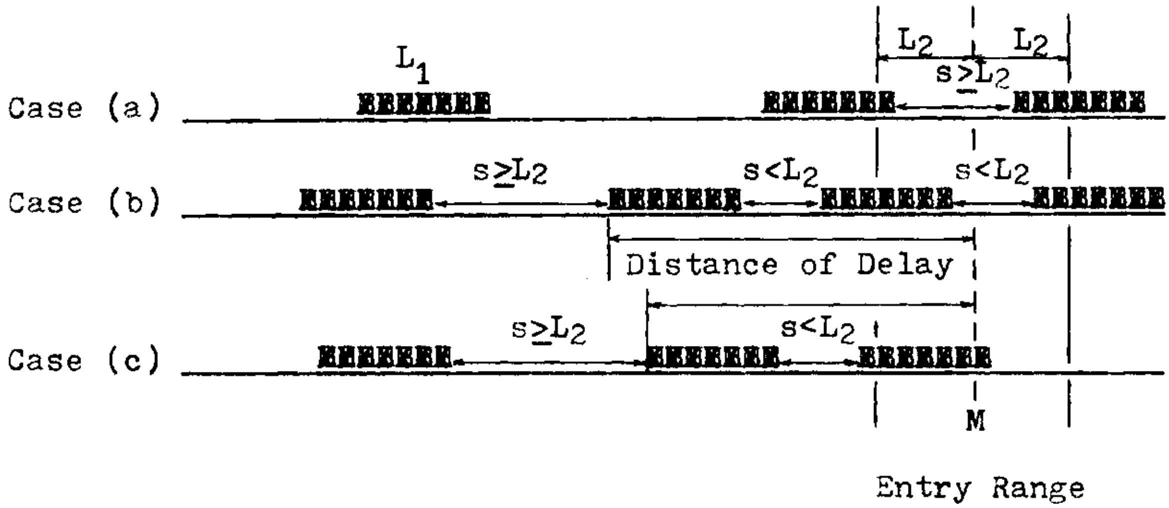

Figure 2-2. Initial Conditions

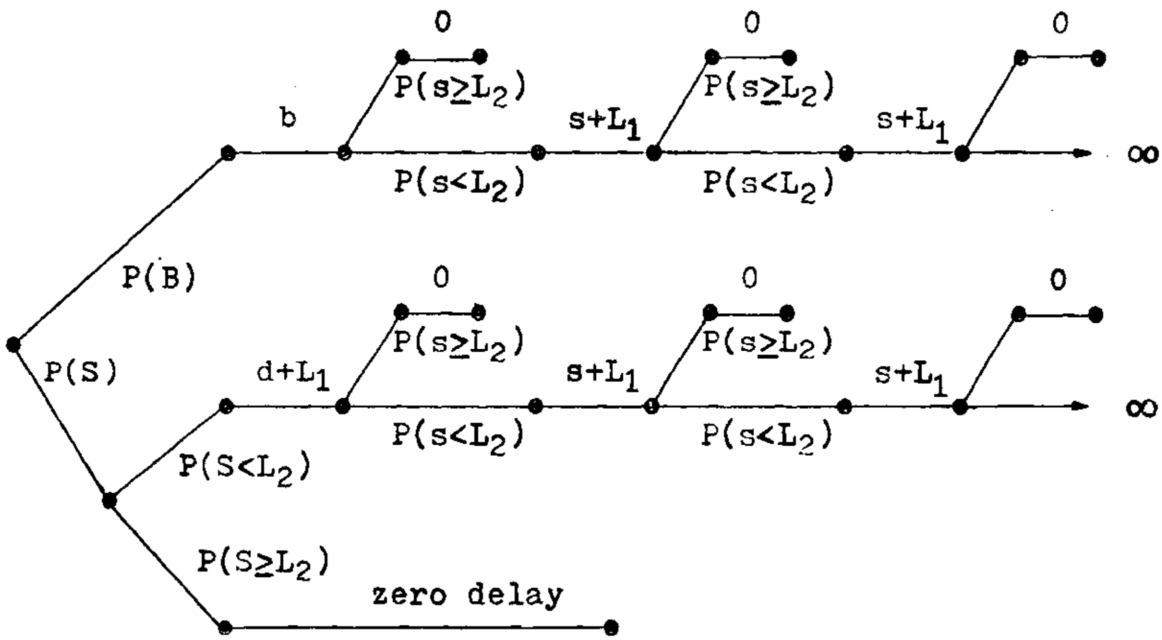

Figure 2-3. Delay Possibilities for an Arrival

or. If this space is a L-space, $s \geq L_2$, the operator will place his load onto the conveyor and go back to work. If this is not true, $s < L_2$, then the operator must wait for that space, s, and the next box, L_1, to go by. Since the operator will wait until the first L-space arrives, this process can continue indefinitely, as shown.

If the operator finds a space initially, P(S), then he may be delayed if that space was a S-space, $S < L_2$. Otherwise he would incur no delay for that attempt. If he is delayed, then he will have to wait for a portion of the S-space, d, and the box following it, L_1, to pass. Once this has happened the operator waits, as before, for the first arrival of a L-space. To show this analytically, let T_B be defined as the delay caused by finding a box initially at M. So,

$$T_B = (b + B + s^*) \frac{1}{v} \qquad 2\text{-}1$$

where b = that portion of the box observed initially that is required to pass the midpoint

B = the cumulative length of a number of boxes required to pass the midpoint before the first L-space occurs

s* = the cumulative length of a number of S-space required to pass before the first L-space occurs

Also, let T_S be defined as an interval of time caused by finding a S-space initially at the midpoint.

Then,

$$T_S = (d + L_1 + B + s^*) \frac{1}{v} \qquad 2\text{-}2$$

where d = a portion of the S-space observed initially between two boxes that is required to pass.

Since T_B and T_S represent the only possibilities for having delay, total delay at the second station, T_2, is

$$T_2 = T_B * P(B) + T_S * P(S) * P(S<L_2) \qquad 2\text{-}3$$

Taking expectations, we have:

$$E[T_2] = P(B)E(T_B) + P(S)P(S<L_2)E[T_S] \qquad 2\text{-}4$$

where, $E[T_B] = (E[b] + E[B] + E[s^*]) \frac{1}{v}$

$$= E[b] \frac{1}{v} + E[B] \frac{1}{v} + E[\frac{s^*}{v}]$$

and $E[T_S] = (E[d] + E[L_1] + E[B] + E[s^*]) \frac{1}{v}$

$$= E[\frac{d}{v}] + \frac{L_1}{v} + E[B] \frac{1}{v} + E[\frac{s^*}{v}]$$

Expressions for P(B), P(S), E(b), E(d), E(B) and E(s*) will now be developed.

Derivation of E(b)

It has been assumed that the operator of the second station will arrive at the midpoint of the entry range randomly. Thus, given that there is initially a box at that point, exactly where on that box the midpoint of the range lies will be described by the uniform distribution. The density function for the uniform is:

$$f(b) = \begin{cases} \frac{1}{L_1 - 0} & \text{for } 0 < b < L_1 \\ 0 & \text{elsewhere} \end{cases}$$

Since the maximum box length is L_1, taking the expectation,

$$E[b] = \int_0^{L_1} b \left(\frac{1}{L_1}\right) db$$

$$= \frac{b^2}{2L_1}\Big]_0^{L_1} \qquad 2\text{-}5$$

$$E[b] = \frac{L_1}{2}$$

<u>Derivation of P(B) and P(S)</u>

We must consider the possibility that arrivals from a station have been unavoidably delayed by initially observing a part of the last box that the station placed on the conveyor. This is referred to as self-blocking, and it reduces the real production rate for a station. To calculate this factor a discussion of station assumptions is in order. First, it has been assumed that for each station there exists an entry range of length $2L_i$. Second, once a box is ready to be placed onto the conveyor it will be placed as close to the center of the entry range as possible. Third, if a box is self-blocked, then it will be "left shifted" by some amount t' on the entry range. This shifting will influence the possibility that, when the next box is finished, it will be self-blocked.

Also, the distribution of time between the arrival of boxes at M is needed. Figure 2-4 represents the time between the leading edges of two boxes on the conveyor before station two.

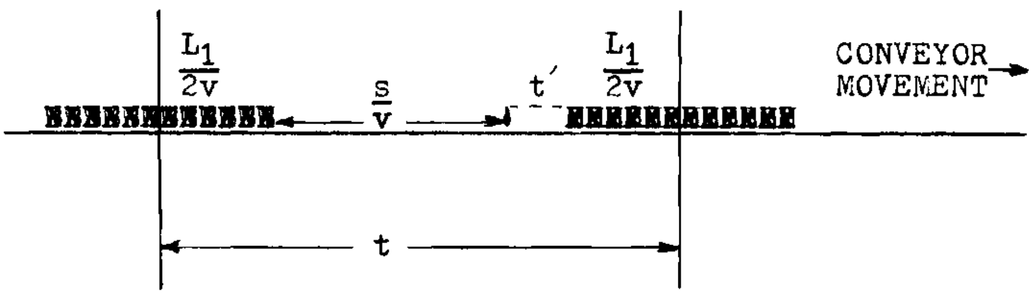

Figure 2-4. The Expected Space Between Boxes

where t = the cycle time of production following the exponential distribution

s = the space between the boxes given in feet

t' = an amount of time attributed to earlier length shifts. It is assumed that the probability of three consecutive left shifts is negligible.

From this figure it can be seen that

$$\frac{s}{v} = \begin{cases} 0 & \text{where} \quad 0 \leq t \leq \frac{L_1}{v} + t' \\ t - (t' + \frac{L_1}{v}) & \frac{L_1}{v} + t' < t < \infty \end{cases}$$

Since t follows an exponential distribution, the probability of finding that the time between boxes is zero, $\frac{s}{v} = 0$ is:

$$P(t) = 1 - e^{-\lambda_1\left(\frac{L_1}{v} + t'\right)}$$

The probability function that describes times between boxes which are greater than zero is:

$$p\left(\frac{s}{v}\right) = \lambda_1 e^{-\lambda_1\left(\frac{s + L_1}{v} + t'\right)} \qquad \text{where } 0 < \frac{s}{v} < \infty$$

Also, the amount of left shift due to self-blocking, t', can be seen to vary in the range of $0 \le t' \le \frac{L_1}{2v}$. This amount can be mathematically described within this range as the following:

1. $t' = \frac{L_1}{2v}$ when $0 \le t \le \frac{L_1}{2v}$ and $P\left(t' = \frac{L_1}{2v}\right) = \left(1 - e^{-\frac{\lambda_1 L_1}{2v}}\right)$
2. $0 < t' < \frac{L_1}{2v}$ when $\frac{L_1}{2v} < t < \frac{L_1}{v}$ and $p(t') = \lambda_1 e^{-\lambda_1\left(t' + \frac{L_1}{2v}\right)}$
3. $t' = 0$ when $\frac{L_1}{v} \le t < \infty$ and $P(t' = 0) = e^{-\frac{\lambda_1 L_1}{v}}$

Now the effect of self-blocking can be calculated. The expected time for self-blocking, $E(t')$, is

$$E(t') = \frac{L_1}{2v}\left(1 - e^{-\frac{\lambda_1 L_1}{2v}}\right) + \int_{\frac{L_1}{2v}}^{\frac{L_1}{v}} t'\lambda_1 e^{-\lambda\left(t' + \frac{L_1}{2v}\right)} dt' + 0\left(e^{-\frac{\lambda_1 L_1}{v}}\right) \qquad 2\text{-}6$$

$$= \frac{L_1}{2v}\left(1 - e^{-\frac{\lambda_1 L_1}{2v}}\right) + e^{-\frac{\lambda_1 L_1}{2v}}\left(e^{-\frac{\lambda_1 L_1}{2v}}\left(\frac{L_1}{2v} + \frac{1}{\lambda_1}\right) - e^{-\frac{\lambda_1 L_1}{v}}\left(\frac{L_1}{v} + \frac{1}{\lambda_1}\right)\right)$$

Thus, the expected self-blocking for any cycle for a given station, E[SB], is

$$E[SB] = \int_0^{\frac{L_1}{2v} + E[t']} t\lambda_1 e^{-\lambda_1 t} dt \qquad 2\text{-}7$$

$$= \frac{1}{\lambda_1} - \left(\frac{L_1}{2v} + E[t'] + \frac{1}{\lambda}\right) e^{-\lambda_1\left(\frac{L_1}{2v} + E(t')\right)}$$

The reduced, self-blocked production rate for the first station, $\bar{\lambda}_1$, becomes:

$$\bar{\lambda}_1 = \frac{\lambda_1}{1 + E[SB]\lambda_1} \qquad 2\text{-}8$$

Having found the reduced output from station 1, we need to know the probability of finding a box on the conveyor. It is stated that P(B) is the probability of finding a box initially at M. Since the operator arrives randomly with a box, the fraction of the conveyor occupied by boxes just prior to the second station is the probability of finding a box initially. Thus,

$$P(B) = \frac{L_1\bar{\lambda}_1}{v} \qquad 2\text{-}9$$

Likewise, P(S) is the probability of not finding a box initially at the midpoint. Thus,

$$P(S) = 1 - P(B) \qquad 2\text{-}10$$

Since the space between boxes arriving at station two depends on the unaffected production rate, λ_1, the probabilities of having a non-zero S-space, $P(0 < s < L_2)$, a S-space, $P(0 \leq s < L_2)$ and a L-space, $P(s > L_2)$ can now be computed.

$$
\begin{aligned}
P(0 < s < L_2) &= P(0 < \frac{s}{v} < \frac{L_2}{v}) \qquad\qquad \text{2-11} \\
&= P(\frac{L_1}{v} + t' < t < \frac{L_2 L_1}{v} + t') \\
&= \int_{t'} P(\frac{L_1}{v} + t' < t < \frac{L_2 + L_1}{v} + t' \mid t')dt' \; P(t') \\
&= \int_{t'} \left[\int_{\frac{L_1}{v} + t'}^{\frac{L_2 + L_1}{v} + t'} \lambda_1 e^{-\lambda_1 t} dt \right] dt' P(t') \\
&= (e^{-\lambda_1 [\frac{3L_1}{2v}]} - e^{-\lambda_1 [\frac{3L_1 + 2L_2}{2v}]})(1 - e^{\frac{-\lambda L_1}{2v}}) \\
&\quad + e^{\frac{-\lambda_1 L_1}{v}} (1 - e^{\frac{-\lambda_1 L_2}{v}}) \int_0^{\frac{L_1}{2v}} e^{-\lambda t} (\lambda e^{-\lambda t' + \frac{L_1}{2v}}) \, dt' \\
&\quad + (e^{\frac{-\lambda_1 L_1}{v}} - e^{\lambda_1 (\frac{L_1 + L_2}{v})}) \, e^{\frac{-\lambda_1 L_1}{v}} \\
&= e^{\frac{-3\lambda_1 L_1}{2v}} - e^{\frac{-\lambda_1 (3L_1 + 2L_2)}{2v}} \\
&\quad + \frac{1}{2} e^{\frac{-3\lambda_1 L_1}{2v}} (1 - e^{\frac{-\lambda_1 L_2}{v}}) \, (1 - e^{\frac{-\lambda_1 L_1}{v}})
\end{aligned}
$$

Since $P(0 \leq s < L_2) = P(0 < s < L_2) + P(s = 0)$, all we need to determine is P(s = 0). Thus,

$$P(s = 0) = P(\frac{s}{v} = 0)$$

$$= P(t \le \frac{L_1}{v} + t')$$

$$= \int_{t'} P(0 \le t \le \frac{L_1}{v} + t' | t')dt'P(t')$$

$$= \int_{t'} \left[\int_0^{\frac{L_1}{v} + t} \lambda_1 e^{-\lambda_1 t} dt \right] dt'P(t')$$

$$= 1-e^{\frac{-3\lambda_1 L_1}{2v}} (1-e^{\frac{-\lambda_1 L_1}{2v}}) \qquad 2\text{-}12$$

$$+ \int_0^{\frac{L_1}{2v}} (1-e^{\frac{-\lambda_1 L_1}{v} + t'}) \lambda_1 e^{-\lambda_1 t' + \frac{L_1}{2v}} dt'$$

$$+ (1-e^{\frac{-\lambda_1 L_1}{v}}) e^{\frac{-\lambda_1 L_1}{v}}$$

$$= 1 - \frac{3}{2} e^{\frac{-3\lambda_1 L_1}{2v}} + \frac{1}{2} e^{\frac{-5\lambda_1 L_1}{2v}}$$

$$P(s \ge L_2) = P(\frac{s}{v} \ge \frac{L_2}{v})$$

$$= P(t \ge \frac{L_2 + L_1}{v} + t')$$

$$= \int_{t'} P(t \ge \frac{L_2 + L_1}{v} + t' | t')dt'P(t')$$

$$= \int_{t'} \left[\int_{\frac{L_2 + L_1}{v} + t'}^{\infty} \lambda_1 e^{-\lambda_1 t}\, dt \right] dt' P(t') \qquad 2\text{-}13$$

$$= e^{-\lambda_1 [\frac{2L_2 + 3L_1}{2v}]} (1 - e^{\frac{-\lambda L_1}{2v}})$$

$$+ e^{-\lambda_1 [\frac{L_1 + L_2}{v}]} \int_0^{\frac{L_1}{2v}} \lambda_1 e^{-\lambda_1 (t' + \frac{L_1}{2v})} e^{-\lambda t'} dt'$$

$$+ e^{-\lambda_1 [\frac{L_1 + L_2}{v} + 0]} (e^{\frac{-\lambda_1 L_1}{v}})$$

$$= e^{-\lambda_1 [\frac{3L_1 + 2L_2}{2v}]} (\frac{3}{2} - \frac{1}{2} e^{\frac{-\lambda_1 L_1}{v}})$$

At this point it is now feasible to find the expected time of any S-space between two boxes, $E[\bar{s}/v]$. It should be noted that any S-space can refer to zero and non-zero lengths between boxes. Also, we are able to find the expected time of a non-zero S-space, $E[\hat{\bar{s}}/v]$. $E[\hat{\bar{s}}/v]$ will be useful in calculating $E(d)$, since this is due to the fact that once a space between boxes is found initially, it must be of some positive quantity. First consider $E[\hat{\bar{s}}/v]$.

$$E[\hat{\bar{s}}/v] = E[0 < s < L_2]$$

$$= E[0 < \frac{s}{v} < \frac{L_2}{v}] \qquad 2\text{-}14$$

$$= \int_0^{L_2/v} \frac{s}{v} \int_{t'} \lambda_1 e^{\lambda_1\left(\frac{s + L_1}{v} + t'\right)} dt' ds P(t') \,/P(0<s<L_2)$$

$$= \{\frac{3}{2} e^{\frac{-\lambda_1 L_1}{2v}} - \frac{1}{2} e^{\frac{-3\lambda_1 L_1}{2v}}\} e^{\frac{-\lambda_1 L_1}{v}} \{\frac{1}{\lambda_1} - \left(\frac{\lambda_1 L_2 + v}{\lambda_1 v}\right) e^{\frac{-\lambda_1 L_2}{v}}\}/P(0<s<L_2)$$

For $E[\bar{s}/v]$,

$$E[0 \leq s \leq L_2] = E[0 < s \leq L_2] P(0 < s < L_2)/P(0 \leq s < L_2)$$

Thus,

$$E[\bar{s}/v] = \{\frac{3}{2} e^{\frac{-\lambda_1 L_1}{2v}} - \frac{1}{2} e^{\frac{-3\lambda_1 L_1}{2v}}\} e^{\frac{-\lambda_1 L_1}{v}} \{\frac{1}{\lambda_1} - \left(\frac{\lambda_1 L_2 + v}{\lambda_1 v}\right) e^{\frac{-\lambda_1 L_2}{v}}\} / P(0 \leq s < L_2) \qquad 2\text{-}15$$

It is also appropriate here to define the probability that the space found initially is a S-space given that a space is at M, $P(S < L_2)$. This probability is defined as the proportion of the expected time of a non-zero S-space, $E[\hat{\bar{s}}/v]$, over the average time between any two boxes, $E[BB] + L_1/v$. These two averages are represented in Figure 2-5.

It should be noted that now the self-blocked mean, $\bar{\lambda}_1$, is being used. E[BB] is seen to be:

$$E[BB] = \frac{1}{\bar{\lambda}_1} - \frac{L_1}{v} \qquad 2\text{-}16$$

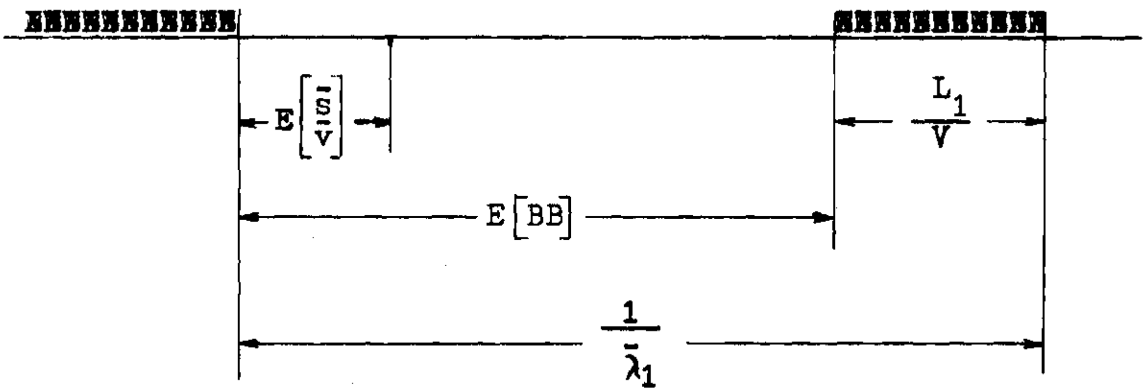

Figure 2-5. Expected Spaces Between Boxes

Also, since this is influenced by the probability of hitting a S-pace, we will have

$$P(S < L_2) = \frac{E[\hat{\bar{s}}/v] * P(0 < s < L_2)}{(E[BB] + L_1/v)} \qquad 2\text{-}17$$

Now having the expected time for the initial S-space observed, the expected time for a portion of such an interval to pass the midpoint of the entry range, E(d), can now be computed. Since the operator is arriving at the entry range randomly, the exact location of the midpoint with respect to the S-space is described by the uniform distribution. Thus, $0 \leq d \leq s$, where d is the portion of the S-space that must pass before the next box is reached. Also, $0 \leq s < L_2$.

To determine this expected delay, we need to compute the following:

$$\frac{1}{v} = \int_{s=0}^{L_2} \int_{d=0}^{s} d f(d|s) P(s) dd ds$$

But $f(d|s) = \frac{1}{s}$. So,

$$E[\frac{d}{v}] = \frac{1}{v} \int_{s=0}^{L_2} \left[\int_{d=0}^{s} \frac{d}{s} \, dd \right] p(s) ds$$

$$= \frac{1}{v} \int_{s=0}^{L_2} \left. \frac{d^2}{2s} \right|_0^s p(s) ds \qquad 2\text{-}18$$

$$= \frac{1}{2} \int_{s=0}^{L_2} \frac{s}{v} \, p(s) ds$$

$$= E(\hat{s}/v)/2$$

Derivation of E[s*/v] and E[B]

It is possible that a number of boxes may pass the operator before an L-space reaches the midpoint of the entry range. By referring to the tree diagram, Figure 2-3, we can find what occurs when a given number of S-spaces and boxes pass the midpoint. After the initial conditions are satisfied, i.e., the passage of the first partial or complete box, the number of boxes passing the midpoint must have been preceded by the same number of S-space. Furthermore, if only a certain number of boxes pass, the last box must be followed by an L-space.

It is known that the length of boxes passing is L_1. Also, on the average, the time for an S-space to pass will be the expected time of any S-space, $E[\overline{s}/v]$. Thus, the expected delay caused by subsequent S-spaces, $E[s^*/v]$, and the expected length due to those boxes following these S-spaces, $E[B]$, can now be found.

$$E[s^*/v] = \sum_{n=0}^{\infty} nE[\overline{s}/v]P(s < L_2)^n P(s \geq L_2) \qquad 2\text{-}19$$

and

$$E[B] = \sum_{n=0}^{\infty} n\ L_1\ P(s < L_2)^n\ P(s \geq L_2) \qquad 2\text{-}20$$

where n is the number of S-space which have occurred.

It is noted that the process described above is similar to the geometric distribtuion in which $P(s < L_2)$ is the probability of a failure, and $P(s \geq L_2)$ is the probability of a success. The operator, once initially delayed, is then faced by a number of geometric trials that could theoretically range from zero to infinity.

Also, the expressions derived above can be reduced by using the following simplifying summation.

$$a\sum_{n=0}^{\infty} n(b)^n = a\frac{b}{(b-1)^2}$$

where a is not dependent on n and $|b| < 1$.

Thus,

$$E[s^*/v] = E[\frac{\bar{s}}{v}]\ P(s \geq L_2) \sum_{n=0}^{\infty} nP(s < L_2)^n$$

$$= E[\frac{\bar{s}}{v}]\ P(s \geq L_2)\ \frac{P(s < L_2)}{[P(s < L_2) - 1]^2}$$

But $P(s < L_2) = (1 - P(s \geq L_2)$.

So,

$$E[\frac{s^*}{v}] = E[\frac{\bar{s}}{v}] \left[\frac{P(s < L_2)}{P(s \geq L_2)} \right] \qquad \text{2-21}$$

Likewise,

$$E[B] = L_1 P(s \geq L_2) \sum_{n=0}^{\infty} nP(s \leq L_2)^n$$

$$= L_1 \left[\frac{P(s < L_2)}{P(s \geq L_2)} \right] \qquad \text{2-22}$$

$E[T_2]$ Defined

From the preceding information it is now possible to formulate the expression for the expected delay at station two. As before,

$$E[T_2] = P(B)E(T_B) + P(S)P(S < L_2)E[T_S]$$

$$= P(B)\ \{E(b)\frac{1}{v} + E[B]\frac{1}{v} + E[\frac{s^*}{v}]\}$$

$$+ P(S)P(S < L_2)\ \{E[\frac{d}{v}] + \frac{L_1}{v} + E[B]\frac{1}{v} + E[\frac{s^*}{v}]\}$$

$$= [\frac{L_1\overline{\lambda}_1}{v}]\{\frac{L_1}{2v} + (\frac{L_1}{v} + E[\frac{s}{v}])\ [\frac{P(s < L_2)}{P(s > L_2)}]\} \qquad 2\text{-}23$$

$$+ [\frac{v - L_1\ \overline{\lambda}_1}{v}]\ \frac{E[\hat{\overline{s}}/v]}{E[BB]}\{\frac{1}{2}E[\frac{\hat{\overline{s}}}{v}] + \frac{L_1}{v} + (\frac{L_1}{v} + E[\frac{\overline{s}}{v}])\ [\frac{P(s < L_2)}{P(s > L_2)}]\}$$

Reduced Output At Station Two

Now having an expression for the expected delay per cycle at station two, its delayed output rate can be determined by the following equation.

$$\frac{1}{\lambda'_2} = \frac{1}{\overline{\lambda}_2} + E[T_2] \qquad 2\text{-}24$$

where $\overline{\lambda}_2$ = the production rate at station two that includes self-blocking, and λ'_2 = the delayed output rate for station two.

To present the station three and station N models, the flow from station two now needs to be expressed mathematically. This flow is made up of a number of boxes from station one and from station two. Obviously, $\overline{\lambda}_1$ represents the mean departure rate of boxes from station one, and λ'_2 represents the mean departure rate of boxes from station two. However, there are three types of boxes departing station two. They are:

1. A box of length L_1
2. A box of length L_2
3. A "box" of length $(L_1 + L_2)$.

Referring to the tree diagram given in Figure 2-3, the mean departure rates for these three types of boxes are:

1. $\tilde{\lambda}_2 = \lambda'_2 P(S)P(S \geq L_2)$ 2-25
2. $\tilde{\lambda}_{12} = \lambda'_2 - \tilde{\lambda}_2$ 2-26
3. $\tilde{\lambda}_1 = \overline{\lambda}_1 - \tilde{\lambda}_{12}$ 2-27

where $\tilde{\lambda}_2$ = the mean departure rate of boxes of length L_2 from station two

$\tilde{\lambda}_{12}$ = the mean departure rate of "boxes" of length $(L_1 + L_2)$ from station two

$\tilde{\lambda}_1$ = the mean departure rate of boxes of length L_1 from station two.

The probability of having these box types on the conveyor can be found by letting:

$P_2(B_1)$ = the probability that at a point just after station two a box of length L_1 is observed.

= the percentage of conveyor taken by a box of length L_1 when departing station two

$$= \frac{\tilde{\lambda}_1 L_1}{v} . \qquad 2\text{-}28$$

$P_2(B_2)$ = the probability that at a point just after station two a box of length L_2 is observed.

= the percentage of conveyor taken by a box of length L_2 when departing station two

$$= \frac{\tilde{\lambda}_2 \, L_2}{v} \, . \qquad 2\text{-}29$$

$P_2(B_{12})$ = the probability that at a point just after station two a "box" of length $(L_1 + L_2)$ is observed.

= the percentage of conveyor taken by a "box" of length $(L_1 + L_2)$ when departing station two.

$$= \frac{\tilde{\lambda}_{12} \, (L_1 + L_2)}{v} . \qquad 2\text{-}30$$

The conditional probability of having one of these three box types to pass a point just after station two, can be found by letting:

$\hat{P}_1$ = the conditional probability of having a box of length L_1 pass a point just after station two.

$\hat{P}_{12}$ = the conditional probability of having a "box" of length $(L_1 + L_2)$ pass a point just after station two.

$\hat{P}_2$ = the conditional probability of having a box of length L_2 pass a point just after station two.

Analytically, this becomes

$$\hat{P}_1 = \frac{P_2(B_1)}{P_2(B_1) + P_2(B_{12}) + P_2(B_2)} \qquad 2\text{-}31$$

$$\hat{P}_{12} = \frac{P_2(B_{12})}{P_2(B_1) + P_2(B_{12}) + P_2(B_2)} \qquad 2\text{-}32$$

$$\hat{P}_2 = \frac{P_2(B_2)}{P_2(B_1) + P_2(B_{12}) + P_2(B_2)} \qquad 2\text{-}33$$

and

$$\hat{P}_1 + \hat{P}_{12} + \hat{P}_2 = 1.$$

The Expected Box Lengths and Arrival Rates at Station 3

Estimates of the probabilities of events occuring immediately after station two on the conveyor are known. In order to use the development of the second station for subsequent stations on the line, the expected box lengths for the three types of boxes must be determined. We know that a number of boxes will be lengths L_1, L_2 and $(L_1 + L_2)$. Since we have the corresponding probabilities are known for each event, the expected value of the box lengths immediately after station two can be expressed as:

$$\hat{L}_2 = \hat{P}_1 L_1 + \hat{P}_2 L_2 + \hat{P}_{12}(L_1 + L_2). \qquad 2\text{-}34$$

Also, the departure rates for the various box lengths are known. The departure rate for all boxes would be the sum of the individual departure mean rates, or

$$\begin{aligned} \hat{\lambda}_2 &= \tilde{\lambda}_1 + \tilde{\lambda}_2 + \tilde{\lambda}_{12} \\ &= \bar{\lambda}_1 + \tilde{\lambda}_2. \end{aligned} \qquad 2\text{-}35$$

Thus, the total conveyor load just after station two can be described by $\hat{\lambda}_2$ and $\hat{L}_2$. Moreover, what departs station two arrives on the conveyor at station three unchanged.

The Three Station Model

This model is similar to the two station model. The third station is characterized by a non-delayed production rate λ_3 and by a box of length L_3. The conveyor load arriving at the third station will have a mean box arrival rate $\hat{\lambda}_2$ and a mean box length of $\hat{L}_2$. The expected delay at station two depends on three parameters:

1. the mean arrival rate of boxes on the conveyor.
2. the length of the boxes arriving at station 2.
3. the length of the box at station 2.

The corresponding value of these parameters at station three will be used in equation 2-23 to estimate $E(T_3)$. This is an approximation since the distribution of the spaces developed for station two will have changed.

The expected delay per cycle due to self-blocking for the third station is found from equations 2-27, and the reduced output from station three becomes

$$\bar{\lambda}_3 = \frac{\lambda_3}{1 + E_3(SB)\ \lambda_3} \ .$$

Now by substituting the appropriate values into $E[T_2]$, the expected delay per cycle for the third station, $E[T_3]$ can be written as:

$$E[T_3] = \frac{\hat{L}_2\hat{\lambda}_2}{v} \ \{\frac{\hat{L}_2}{2v} + (\frac{\hat{L}_2}{v} + E[\frac{\bar{s}}{v}]) \ [\frac{P(s < L_2)}{P(s > L_2)}]\}$$

$$+ \left[\frac{v - \hat{L}_2\hat{\lambda}_2}{v}\right] \frac{E[\hat{\bar{s}}/v]}{E[BB]} \left\{\frac{1}{2} E[\frac{\hat{s}}{v}] + \frac{\hat{L}_2}{v} + \left(\frac{\hat{L}_2}{v} + E[\frac{\bar{s}}{v}]\right)\right.$$ 2-36

$$\left.\left[\frac{P(s < L_2)}{P(s > L_2)}\right]\right\}$$

The Reduced Output at Station Three

Likewise, the expected delayed output rate at station three, λ'_3, is now

$$\frac{1}{\lambda'_3} = \frac{1}{\bar{\lambda}_3} + E[T_3]$$ 2-37

The departure rates from station three can now be found. As before, there are three types of "boxes". They are:

1. A box of length L_3
2. A "box" of length $(L_3 + \hat{L}_2)$
3. A "box" of length $\hat{L}_2$

While these "boxes" may be composed of a box of length $L_3 + L_1$, $L_3 + L_2$, $L_3 + L_2 + L_1$, $L_2 + L_1$, L_2 and L_1, the reason for placing them into these separate types is to be able to predict:

1. A non-delayed box loaded at the third station
2. A delayed box loaded at the third station
3. A "box" arriving on the conveyor at the third station and not having its length increased.

Thus, the departure rates for these three types of boxes become:

$$\tilde{\lambda}_3 = \lambda'_3 \, P(S)P(S \geq L_3)$$

$$\tilde{\lambda}_{\hat{2}3} = \lambda'_3 - \tilde{\lambda}_3 \qquad \text{2-38}$$

$$\tilde{\lambda}_{\hat{2}} = \hat{\lambda}_2 - \hat{\lambda}_{\hat{2}3}$$

where $\tilde{\lambda}_3$ = the departure rate of a box of length L_3 from the third station

$\tilde{\lambda}_{\hat{2}3}$ = the departure rate of a "box" of length $(\hat{L}_2 + L_3)$ from station three

$\tilde{\lambda}_{\hat{2}}$ = the departure rate of a "box" of length $\hat{L}_2$ from station three.

The probability of having these three "box" types on the conveyor can be found by letting:

$P_3(B_3)$ = the probability that at a point just after station three a box of length L_3 is observed

$$= \frac{\tilde{\lambda}_3 L_3}{V} \,. \qquad \text{2-39}$$

$P_3(B_{\hat{2}3})$ = the probability that at a point just after station three a "box" of length $L_3 + \hat{L}_2$ is observed.

$$= \frac{\tilde{\lambda}_{\hat{2}3} \; (\hat{L}_2 + L_3)}{V} \,. \qquad \text{2-40}$$

$P_3(B_{\hat{2}})$ = the probability that at a point just after station three a "box" of length $\hat{L}_2$ is observed.

$$= \frac{\tilde{\lambda}_{\hat{2}}\ \hat{L}_2}{v} \quad . \tag{2-41}$$

The conditional probabilities of having one of these three box types to occur after station three can be written analytically as:

$$\hat{P}_3 = \frac{P_3(B_3)}{P_3(B_3) + P_3(B_{\hat{2}3}) + P_3(B_{\hat{2}})} \quad . \tag{2-42}$$

$$\hat{P}_{\hat{2}3} = \frac{P_3(B_{\hat{2}3})}{P_3(B_3) + P_3(B_{\hat{2}3}) + P_3(B_{\hat{2}})} \quad . \tag{2-43}$$

$$\hat{P}_{\hat{2}} = \frac{P_3(B_{\hat{2}})}{P_3(B_3) + P_3(B_{\hat{2}3}) + P_3(B_{\hat{2}})} \quad . \tag{2-44}$$

Using the above, the expected length of boxes departing station three, $\hat{L}_3$, becomes:

$$\hat{L}_3 = \hat{P}_{\hat{2}}\hat{L}_{\hat{2}} + \hat{P}_{\hat{2}3}(\hat{L}_{\hat{2}} + L_3) + \hat{P}_3 L_3 . \tag{2-45}$$

Also, the expected rate at which a "box" departs from station three, $\hat{\lambda}_3$, is:

$$\hat{\lambda}_3 = \hat{\lambda}_2 + \tilde{\lambda}_3 . \tag{2-46}$$

Thus, we have once again defined the departing conveyor load from a station. This in turn will define what will arrive at the next station. By making the proper substitutions into the previous station model, the departing conveyor loads can sequentially be determined.

The N Station Model

The above procedure is continued iteratively until the last station on the conveyor is reached. It is proposed here that by making the appropriate substitutions of various parameters into the earlier station models, the N station model can be developed as the third was.

The Nth station is characterized by a non-delayed production rate, λ_N, and by a box of length L_N. The conveyor load arriving at this station is approximated by a rate, $\hat{\lambda}_{N-1}$, and a mean "box length", $\hat{L}_{N-1}$. By substituting $\hat{\lambda}_{N-1}$, $\hat{L}_{N-1}$, λ_N and L_N for $\overline{\lambda}_1$, L_1, λ_2 and L_2 respectively in the station two model, the expected delay for the Nth station can be derived.

The expected delay per cycle due to self-blocking for the Nth station is found by equation 2-27, and the reduced output from this station becomes

$$\overline{\lambda}_N = \frac{\lambda_N}{1 + E_N(EB)\lambda_N} \qquad 2\text{-}47$$

By substituting the appropriate values into $E[T_2]$, the expected delay per cycle for the Nth station, $E[T_N]$, can now be written as:

$$E[T_N] = [\frac{\hat{L}_{N-1}\hat{\lambda}_{N-1}}{v}] \; \{\frac{\hat{L}_{N-1}}{2v} + (\frac{\hat{L}_{N-1}}{v} + E[\frac{\overline{s}}{v}]) \; [\frac{P(s < L_2)}{P(s \geq L_2)}]\}$$

$$+ \left[\frac{v-\hat{L}_{N-1}\hat{\lambda}_{N-1}}{v}\right] \frac{E[\hat{\bar{\frac{s}{v}}}]}{E[BB]} \left\{\frac{1}{2}E[\hat{\bar{\frac{s}{v}}}] + \frac{\hat{L}_{N-1}}{v}\right.$$

$$\left. + \left(\frac{\hat{L}_{N-1}}{v} + E[\bar{\frac{s}{v}}]\right) \left[\frac{P(s < L_2)}{P(s \geq L_2)}\right]\right\} \qquad 2\text{-}48$$

The reduced output rate at station N becomes

$$\frac{1}{\lambda'_N} = \frac{1}{\bar{\lambda}_N} + E[T_N]. \qquad 2\text{-}49$$

By using the same arguments as in the third station model, the departure rates from the Nth station become:

$$\tilde{\lambda}_N = \lambda'_N P(S) P(S \geq L_N) \qquad 2\text{-}50$$

$$\tilde{\lambda}_{\hat{N}-1,N} = \lambda'_N - \tilde{\lambda}_N \qquad 2\text{-}51$$

$$\tilde{\lambda}_{\hat{N}-1} = \hat{\lambda}_{N-1} - \hat{\lambda}_{\hat{N}-1,\ N} \qquad 2\text{-}52$$

The probability of having these three "box" types on the conveyor is:

$$P_N(B_N) = \frac{\tilde{\lambda}_N L_N}{v} \qquad 2\text{-}53$$

$$P_N(B_{\hat{N}-1,\ N}) = \frac{\tilde{\lambda}_{\hat{N}-1,N}\ (\hat{L}_{N-1} + L_N)}{v} \qquad 2\text{-}54$$

$$P_N(B_{\hat{N}-1}) = \frac{\tilde{\lambda}_{\hat{N}-1}\ \hat{L}_{N-1}}{v} \qquad 2\text{-}55$$

The conditional probability of having one of these box types occur after station N can be described analytically as:

$$\hat{P}_N = \frac{P_N(B_N)}{P_N(B_N) + P_N(B_{\hat{N}-1,N}) + P(B_{\hat{N}-1})}. \tag{2-56}$$

$$\hat{P}_{\hat{N}-1,\ N} = \frac{P_N(B_{\hat{N}-1,N})}{P_N(B_N) + P_N(B_{\hat{N}-1,N}) + P(B_{\hat{N}-1})} \tag{2-57}$$

$$\hat{P}_{\hat{N}-1} = \frac{P_N(B_{\hat{N}-1})}{P_N(B_N) + P_N(B_{\hat{N}-1,N}) + P(B_{\hat{N}-1})} \tag{2-58}$$

The expected length of boxes departing station N, $\hat{L}_N$, can be written as:

$$\hat{L}_N = \hat{P}_{\hat{N}-1}\hat{L}_{N-1} + \hat{P}_{\hat{N}-1,N}(\hat{L}_{N-1} + L_N) + \hat{P}_N L_N \tag{2-59}$$

The expected rate at which a "box" departs from station N, $\hat{\lambda}_N$, can be described by:

$$\hat{\lambda}_N = \hat{\lambda}_{N-1} + \tilde{\lambda}_N\ . \tag{2-60}$$

Thus, by using the models developed in this chapter, the expected delays and the delayed output rates for a number of stations can be calculated for a given set of box lengths and non-delayed production rates. The next chapter will present a simulation model for such a system.

CHAPTER III

SIMULATION MODEL DEVELOPMENT

A simulation model was developed with the intent of investigating the analytical model's performance. The model is event oriented, hence, information is maintained for every box placed on the conveyor. The simulator first tabulates the arrival times of boxes from station one. After tabulating these times, the boxes from the other stations are entered singly. The remainder of this chapter presents the structure of the simulator. A complete listing of the fortran code can be seen in Appendix (A-1).

The overall structure of the simulator model is depicted in the flow diagram of Figure 3-1. The model can be segmented into four broad areas. These areas are Initialization, Generation of Boxes from the First Station, Generation of Boxes from the Other Stations, and System Output.

Initialization

In the first segment a number of parameters are read into the simulator, and a number of vectors are initialized. The parameters that are read are:

1. the number of stations,
2. the conveyor speed,

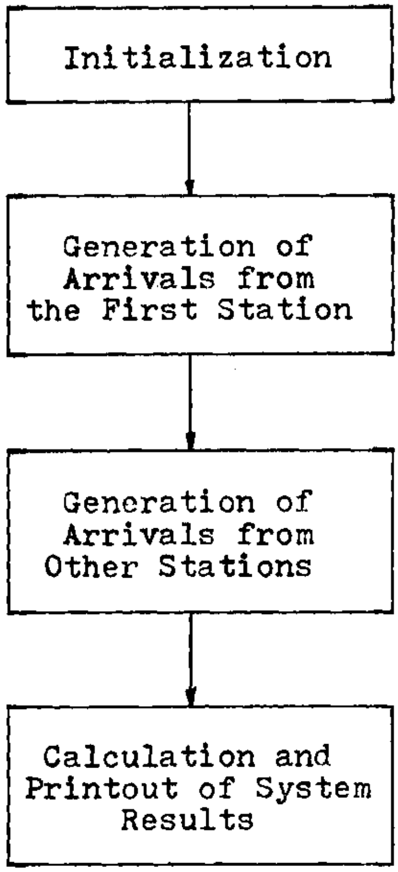

Figure 3-1. Overall Structure of Simulator

3. the "hours" of observation,
4. the non-delayed production rates,
5. the box lengths.

The vectors that are initialized are:

1. an array for recording delay times,
2. an array for recording the number of attempts delayed,
3. an array for recording self-blocking,
4. an array for recording the number of total loading attempts,
5. an array for recording the position of boxes on the conveyor.

Also, the number of boxes from the first station is computed. A flow diagram of the first segment can be seen in Figure 3-2.

Generation of Boxes from Station One

In this segment the conveyor positions of boxes from the first station are generated. The process for finding the positions starts at time zero when a box is placed in the middle of the entry range. The entry range of two box lengths is used. Also, an allowance for self-blocking is incorporated into the simulation by the following method. If a box has not passed from the entry range sufficiently when the next arrival is generated for this station, then the operator will be delayed unavoidably until the box can

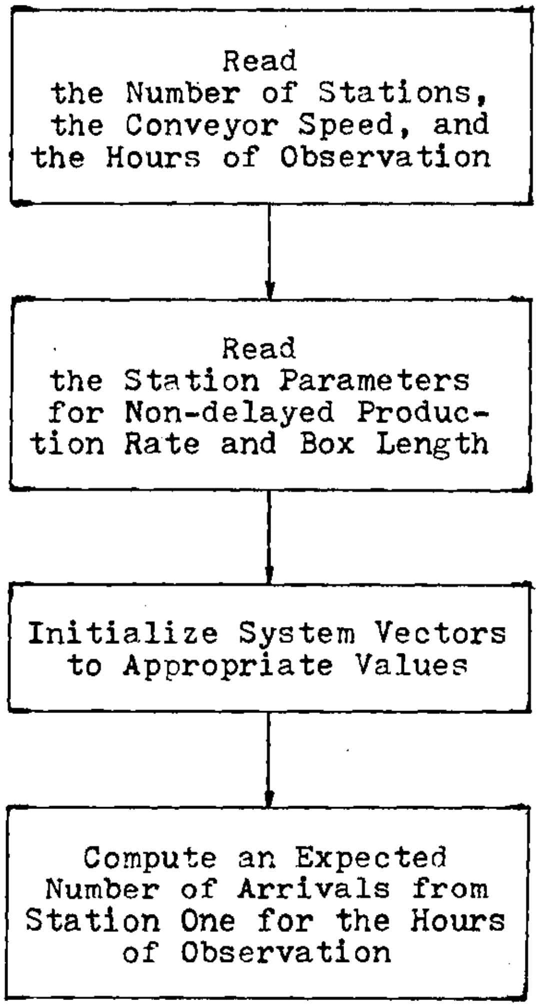

Figure 3-2. Initialization

be placed within the entry range. The first station can never by delayed in any other way.

Each box loaded at a station will be placed as close to the middle of the entry range, "M", as possible. The position on the conveyor of each box is identified by the time of each leading edge, i.e., the rightmost edge of the box when the conveyor is moving to the right. Also, the length of a box and the station from which a box was placed onto the conveyor are recorded into other arrays. This basic process is repeated until the last specified box from station one has been placed onto the conveyor. The leading edge of the last box is then set equal to the "span of time" for which the observations of boxes from the other stations will take place. A flow diagram of the second segment can be seen in Figure 3-3.

Generation of Boxes from Other Stations

The previous section of the simulator has now positioned a number of boxes from the first station on the conveyor. This section will handle the positioning of boxes on the conveyor from subsequent stations. The process for placing boxes onto the conveyor begins with the second station at time zero. Each box from this station is treated individually. Also, if the box has been delayed, the time of that delay is recorded when the box has been positioned on the conveyor. When no additional boxes from the second

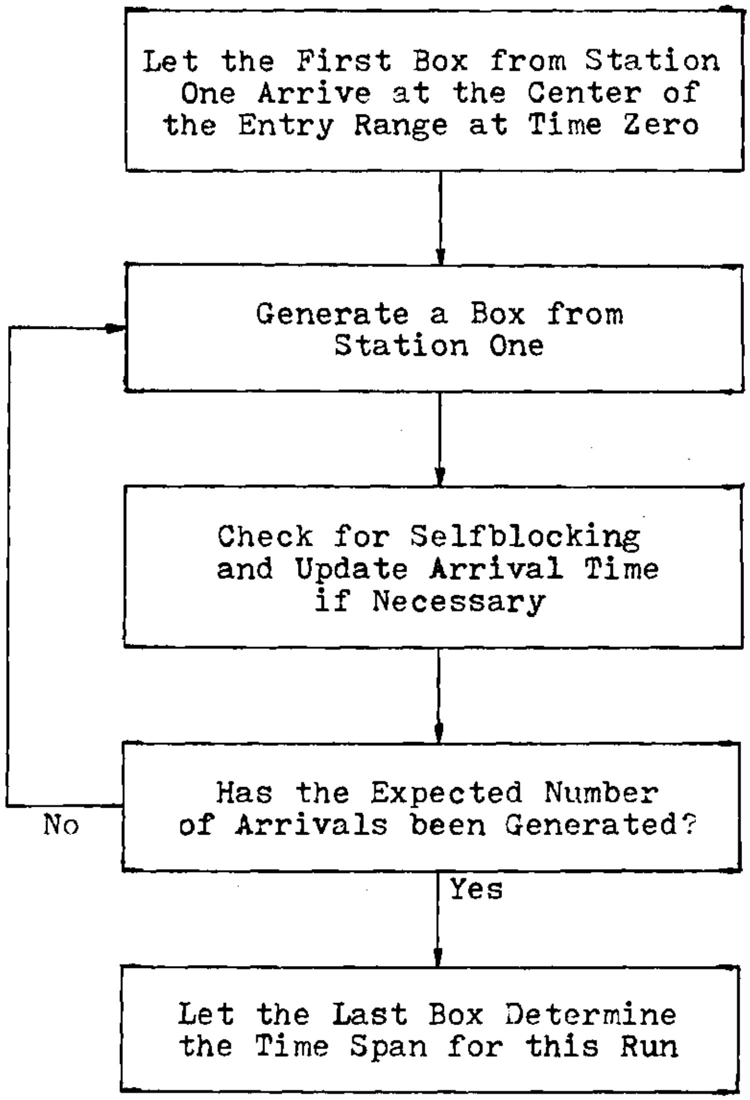

Figure 3-3. Generation of Boxes from Station One

station can be placed onto the conveyor, the process will start over at time zero by observing arrivals from station three. Once the boxes are positioned, the process will be repeated until the last station has positioned all of its boxes onto the conveyor. The flow diagram for this process can be seen in Figure 3-4. This process is described in greater detail in the following discussion.

For a given station, the process starts by generating an arrival time for a box by using an exponential routine. The mean of the exponential distribution used for this generation procedure is described by the reciprocal of the station's mean non-delayed production rate. Once an arrival is generated, the box is checked for self-blocking with the last arrival from that station. If it is found that there has been insufficient time for that box to pass from the entry range, the arrival time for the box is increased until the last box has been sufficiently removed. This increase is what is referred to as self-blocking. Also, the process retains all the information concerning the station prior to the arrival of the new box. It will not include the new arrival until it has been placed upon the conveyor within the allotted time span. If it is found that the box can not be placed on the conveyor within the time span, the information derived for that box alone will not be considered for finding estimates of the station's performance.

For each arrival at the midpoint the entry range, "M",

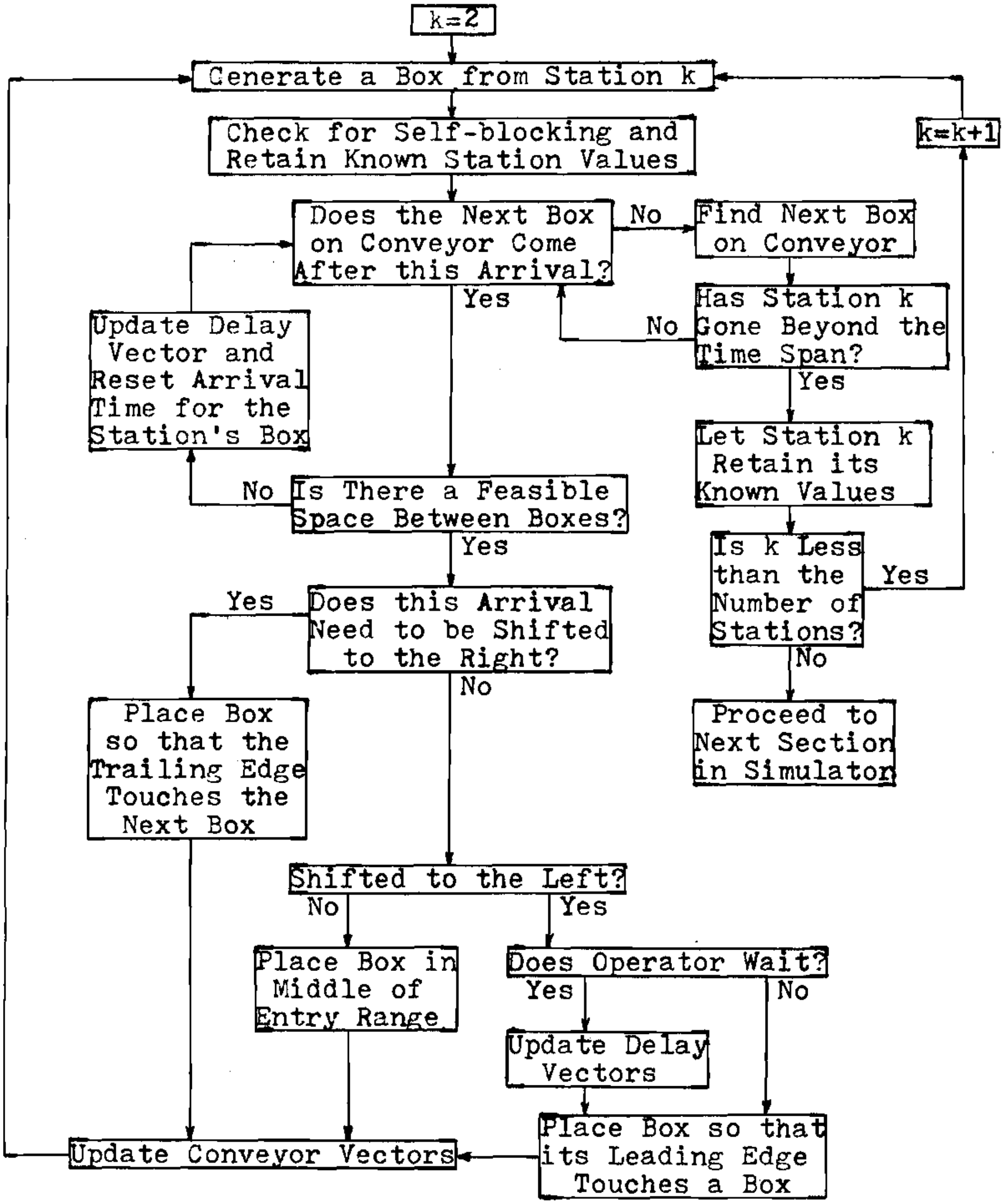

Figure 3-4. Generation of Station k Boxes

for a given station, its relative position, in time, is found on the conveyor with respect to leading box edges. This procedure is accomplished by first searching the conveyor for the times of leading edges. The search will end once the time of a box's leading edge is found to be greater than the arrival time. Thus, the box arrives at "M" between this box on the conveyor and the one just before it. If the space between these two boxes on the conveyor is found to be too small for entering, then a delay is recorded for the station, and its arrival time is increased to allow one box on the conveyor to pass. The process will start a new search and will continue until its relative position on the conveyor is found.

If it had been found that there is a feasible space between the two boxes, the process determines how to position the box within the station's entry range. The first question asked is whether the box must be positioned in the right side of the entry range, so that the following box is not interferred with. If it is necessary to do so, then the box will be placed appropriately on the conveyor. Otherwise, another question will be raised. Since we now know that there is a feasible space between boxes, and the arrival of this box does not interfere with the following, "upstream" box, what interference is caused by the "downstream" box? If none, the box is placed in the center of the entry range. However, if it is found that there is interference in placing the box within the entry range, the question of the magnitude of the interference is asked. If the interference is sufficiently qreat, then the operator will be delayed until half of the entry range is emptied. Otherwise, there will be no delay. In both cases, the box will be positioned directly behind the "downstream" box. Thus, in such a manner, a box from a station is entered onto the conveyor. Once a box is placed onto the conveyor, the program updates the other conveyor vectors, in regard to where the new arrival is positioned, and returns the process to that point where a new box from the station is to be generated.

If a new box is found to go beyond the established time span, then the information concerning that solution prior to the newest arrival is retained. The simulator checks to see if the next station needs to be investigated. If so, the first box from the next station is generated and the process starts over. However, if it is found that all stations have been investigated, the simulator proceeds to the next section.

System Output

The last segment reports the results of the simulation run, and its flow diagram can be seen in Figure 3-5. For each station the number of boxes placed on the conveyor, the total time of delay, the number of boxes delayed and the percent of delay are given. The average delay, the total

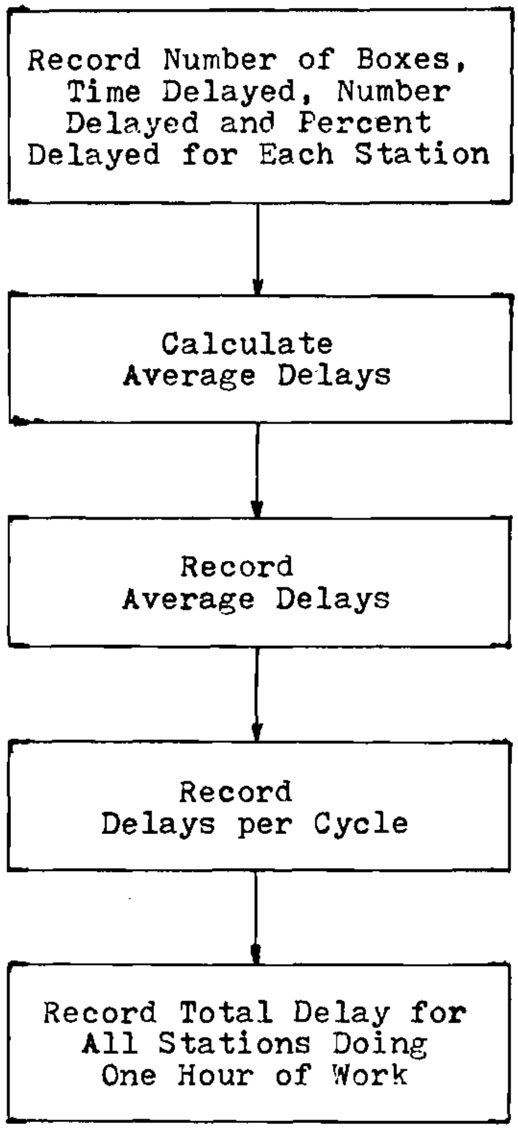

Figure 3-5. System Output

delay time divided by the number placed on the conveyor, is also calculated for each station. This amount is then recorded as the average delay per cycle of a station. The average delays per cycle are multiplied by their respective mean station arrival rates. This product is the total expected delay for that station performing one hours worth of work. The values are then summed by a total expected delay for all stations performing one hour of work. This sum is what is referred to in the next chapter as arrangement delay. All of the values are then printed out. The use of this model in investigating the analytic model and a number of stations is reported in the following chapters.

CHAPTER IV

MODEL RESULTS

In this chapter comparisons of the models developed in Chapters II and III are given, and the heuristic station arrangement strategies presented later in this chapter are evaluated.

Comparison of Models

The simulation and analytical models were compared to one another using a number of systems which are described by several parameters. Once the values of these parameters have been specified, the system will be referred to as a set of stations or simply, a set. The parameters needed are the number of stations in series; the conveyor speed; the mean, non-delayed production rate for each station; the length of boxes loaded at each station; and the "initial" time for observing the system.

The number of stations was selected to be four. The basic reason for this selection was that, intuitively, differences between these models should begin to appear at either the second or third station. This would appear to be the case due to the distributions of times between boxes on the conveyor after the second station. Referring to Chapter II, the analytic model approximates these subsequent distributions as exponential, while in the simulator they may be of a different form.

The conveyor speeds were selected to be 2400, 4800 and 7200 feet per hour. From a manual investigation of movement over a length of ten feet, it appeared that anything below 2400 feet per hour probably would be too slow for this preliminary study due to the self-blocking and other delays that would occur. Also, anything beyond 7200 feet per hour would hinder the loading operation in practical situations. In the preliminary investigation all four stations were required to have the same non-delayed production rate, and the boxes to be loaded at each station were of the same length. Production rates of 30, 100, 300, and 600 boxes per hour were used. Also, the box length was held at one foot for all stations.

The length of the simulation run was selected so that approximately 2400 boxes would be loaded onto the conveyor if there were no delays. Thus, the lengths of the simulated production times were 20, 6, 2, and 1 hours for the production rates of 30, 100, 300, and 600 boxes per hour, respectively.

The simulator was run five times, using different seeds for the psuedo random number generator for each set of parameters. The average time for delay per station and its standard deviation obtained from these runs are presented in Table 4-1. Also, using these same sets, the values for the expected delay for each station, E[T], have been computed using the analytic model. The results are also shown in Table 4-1. In Table 4-1, as in subsequent tables, the notation, 30/1, describes a station having a mean rate of 30 boxes per hour and a box length of one foot. Also, station one is not listed since it does not experience delays due to boxes coming from other stations.

In Table 4-1 the figures presented under the heading "Delay Time in Hours" are the delays found in the simulator and the analytic models. The entries under the "Simulator Model" heading are averages of the five replications, that is, the sum of the five observations for each station divided by five. The differences between the models are reported, both in delay in 10^{-6} hours and in terms of the sample standard deviation.

From Table 4-1, it can be observed that both models behave similarly in many cases. In other cases, the analytical model understates the value of delay for the first station. However, using the sets of stations with higher production rates, it can be seen that the analytical model overestimates the values that were computed by the simulator. Also, these overestimates are the times greater than three sample standard deviations. In observing the simulator results alone, the sample standard deviation for the five replicates appears to be tighter for higher production rates. This would suggest that in order to reduce the standard de-

Table 4-1. A Comparison of Simulator and Analytic Models

			Delay in Hours*10^{-6}			Differences	
Rate /Box	Speed ft/hr	Sta- tion	Simu- lator	Std. Dev. (s)	Ana- lytic	Rela- tive	in Terms of (s)
30/1	2400	2	2	0.8	3	1	1.25
		3	4	2.2	6	2	0.91
		4	10	4.3	11	1	0.23
	4800	2	1	0.5	1	0	0.00
		3	1	0.5	1	0	0.00
		4	2	0.5	2	0	0.00
	7200	2	0	0.4	0	0	0.00
		3	1	0.5	1	0	0.00
		4	1	0.4	1	0	0.00
100/1	2400	2	11	2.9	11	0	0.00
		3	34	4.6	31	-3	-.65
		4	53	6.7	63	10	1.49
	4800	2	2	0.5	2	0	0.00
		3	6	1.2	6	0	0.00
		4	10	1.5	11	1	0.67
	7200	2	1	0.4	1	0	0.00
		3	2	0.4	2	0	0.00
		4	4	1.1	4	0	0.00
300/1	2400	2	51	4.3	48	-3	-.70
		3	173	19.4	199	26	1.34
		4	385	23.6	527	142	6.02
	4800	2	10	0.0	9	-1	----
		3	29	3.3	29	0	0.00
		4	50	1.8	65	15	8.33
	7200	2	4	0.8	4	0	0.00
		3	10	2.0	10	0	0.00
		4	17	1.3	21	4	3.08
600/1	2400	2	172	13.1	147	-25	-1.91
		3	676	47.5	786	110	2.32
		4	1981	122.8	2061	80	0.65
	4800	2	26	2.0	24	-2	-1.00
		3	87	9.8	99	12	1.22
		4	192	11.8	264	72	6.10
	7200	2	10	1.2	9	-1	-.83
		3	29	1.5	32	3	2.00
		4	56	1.1	76	20	18.18

viation for the lower production rates either more boxes on the conveyor would need to be observed for a given run or a greater number of replicates need to be made. Also, the average time required on the computer to compute these values was approximately 1.8 minutes for a given speed.

From the above discussion it appears that the models are not in total agreement at the microscopic or station level. If this had been the case, it would have been possible to use the analytical model alone in finding the station delays in subsequent investigations. However, it should be noted that, while the purpose of the preliminary investigation was to observe the nature of differences between the models for each station, the scope of the research is to obtain information on the arrangement of stations along a conveyor. Thus, the comparison of individual station differences is inadequate for finding this information.

One measure of interest is the "total delay from all stations, in a given arrangement or set, doing an hours worth of work." This value will be referred to as "arrangement delay" and is computed by the following procedure. The delay per box for a given station is found by a model and is then multiplied by that station's non-delayed production rate. This procedure is used to calculate the "station delay per one hour of work" for every station in a given set. Once all of the station values have been computed, they are summed. The summed value is what is referred to as "arrangement delay". In using arrangement delay as a decision criterion, two assumptions have been made:

1. The cost due to delay at each station is of equal value.
2. The cost due to non-delay factors, such as costs in having a station placed in one particular position, can be ignored.

The values in Table 4-1 have been recalculated as described above and are presented in Table 4-2. Also, values for the sample standard deviation of the arrangement delays, "d", have been included into Table 4-2. Again, for certain cases both models appear to be similar. However, as the station's production rates increase, it appears that these differences between the models increase. From this preliminary investigation, the following decisions were made:

1. While both models predict delays which are nearly the same at times, the use of the analytical model alone in this investigation of station arrangement can not be justified. Thus, it was decided to use both models and observe the results of each.

2. The variability of the results obtained from the simulator model needs to be reduced, but there existed a penalty for excessive usage of computer time. Thus, it was decided that one run of a total expected number of 9000 entries would be appropriate for any given set of parameters.

Table 4-2. A Preliminary Comparison of Simulator and Analytic Models Using Arrangement Delay

Rate /Box	Speed ft/hr	Arrangement Delay in Hours*10^{-6} Simu-lator (D)	Std. Dev. (d)	d/D	Ana-lytic	Differences in Terms of (d)
30/1	2400	480	133	0.28	600	0.9
	4800	120	25	0.21	120	0.0
	7200	60	34	0.57	60	0.0
100/1	2400	9800	992	0.10	10500	0.7
	4800	1800	195	0.11	1900	0.5
	7200	700	114	0.16	700	0.0
300/1	2400	182700	11078	0.06	232200	4.4
	4800	26700	1417	0.05	30900	2.9
	7200	9300	1004	0.11	10500	1.2
600/1	2400	1697400	98740	0.06	1796400	1.0
	4800	183000	11345	0.06	232200	4.3
	7200	57000	1368	0.02	70200	9.6
Sum Total d/D				1.79		
Estimated Standard Deviation in Terms of a Percent				0.15		

The usage of one replicate does not allow for minute observations of the system. Rules describing the ordering of work stations can now only be justified in terms of major changes or patterns occurring in a system. An approximate standard deviation can be computed from the "estimated standard deviation in terms of a percent" found in Table 4-2. One could rougly approximate the standard deviation for 9000 entries by dividing 0.15 by the square root of 9000 over 2400. Thus, the value for the standard deviation could be approximated as 0.08 for 9000 entries.

Heuristic Station Arrangement Strategies

As mentioned briefly in the last section, the overall objective of the research is the development of work station arrangement strategies that reduce production costs. The costs used in this investigation are only associated with the time of delay. Also, delay costs for each station in series are assumed to be equal. Thus, the arrangement delay is an appropriate criterion to use in making comparisons.

In specifying how to order stations along a conveyor, certain station characteristics need to be defined. In this study individual stations have been defined by a production rate and a box length. Thus, simple arrangement rules will be developed in terms of these parameters. In this light, the following work rules are investigated.

Test 1

Given a constant box length and having the first station placed so that it is never delayed by a box from the other stations:

1. Arrange stations in terms of increasing production rates:

2. Arrange stations in terms of decreasing production rates;

3. Arrange stations randomly.

Test 2

Given a constant production rate and having the first station placed so that it is never delayed by a box from the other stations:

1. Arrange stations in terms of increasing box length;

2. Arrnage stations in terms of decreasing box length;

3. Arrange stations randomly.

In the next test, P, the product of the production rate and the box length for a station, is used to describe the arrangement of stations. Also, rules concerning tie breaking are specified.

Test 3

Given a mixture of box lengths and production rates:

1. Arrange stations by increasing P and break ties by selecting the smallest production rate first.

2. Arrange stations by increasing P and break ties by selecting the largest production rate first.

3. Arrange stations by decreasing P and break ties by selecting the smallest production rate first.

4. Arrange stations by decreasing P and break ties by selecting the largest production rate first.

5. Arrange stations randomly.

6. Arrange stations in terms of increasing rates and break ties by ordering stations by increasing length.

In using these work strategies, two separate groups of stations, involving ranges of production rates and box lengths, were observed. The first group had production rates of 30, 90, and 150 boxes per hour and box lengths of 1, 2, 3, and 4 feet. The second group had production rates of 300 and 600 boxes per hour and box lengths of 1/2 and 1 feet. The first group consisting of ten stations was tested for speeds of 2400, 4800, and 7200 feet per hour. However, due to the high utilization of the conveyor for a speed of 2400 feet per hour, the second group, consisting of six stations, was tested over the speeds of 4800 and 7200 feet per hour. In the strategies involving constant parameters, test 1 and test 2, two given values were initially selected for each group. Also, each computer run for a given number of stations placing approximately 9000 boxes onto the conveyor used four minutes on the average.

Test 1 - Constant Box Length

The results in "arrangement delay" of the investigation of strategies invovling a constant box length are given in Tables 4-3 and 4-4 and are summarized in Table 4-5. In order to statistically investigate significant differences in strategies, Friedman's multi-sample test was used. The null hypothesis for Friedman's test states that treatments will have the same effects and, thus, the simulator rankings of strategies in Table 4-5 should be in random order for both groups. By using an α-value of 0.10, it was found that the null hypothesis can be rejected by both groups. Thus, the rejection of the null hypothesis indicates that it will be more desirable to employ the slowest station first policy than the other strategies observed in Test 1. Also, the simulator and the analytic model can be compared here. While the individual values for both models are not exactly the same in Tables 4-3 and 4-4, the ranking of strategies on Table 4-5 are similar.

Test 2 - Constant Production Rates

The results in "arrangement delay" of the investigation of strategies involving a constant production rate are given in Tables 4-6 and 4-7 and are summarized in Table 4-8. As in Test 1, Friedman's multi-sample test was used to analyze the summarized data in Table 4-8. It was found that the null hypothesis can not be rejected at an α-value of 0.10 for the group of stations having the slower production rates of 30, 90, and 150. However, the null hypothesis was rejected for the

Table 4-3. Arrangement Delay in Hours for Constant Box Length Strategies

Strategy: Slowest Station First

Station	1	2	3	4	5	6	7	8	9	10
Rate	30	30	30	90	90	90	90	150	150	150

Length=1 Speed	2400	4800	7200
Simulator	0.101711	0.014103	0.005123
Analytic	0.150223	0.019151	0.006505
Length=3			
Simulator	4.542593	0.342695	0.098727
Analytic	4.536817	0.559758	0.150223

Strategy: Fastest Station First

Station	1	2	3	4	5	6	7	8	9	10
Rate	150	150	150	90	90	90	90	30	30	30

Length=1 Speed	2400	4800	7200
Simulator	0.105482	0.014500	0.005193
Analytic	0.154851	0.019751	0.006673
Length=3			
Simulator	9.819713	0.385545	0.105482
Analytic	4.555279	0.571432	0.154851

Strategy: Random

Station	1	2	3	4	5	6	7	8	9	10
Rate	90	150	30	30	90	90	90	30	150	150

Length=1 Speed	2400	4800	7200
Simulator	0.098899	0.014925	0.005410
Analytic	0.149873	0.019251	0.006542
Length=3			
Simulator	5.815062	0.344083	0.098899
Analytic	4.671987	0.555353	0.149873

Table 4-4. Arrangement Delay in Hours for Constant Box Length Strategies

Strategy: Slowest Station First

Station	1	2	3	4	5	6
Rate	300	300	300	600	600	600
Length=½ Speed			4800			7200
Simulator			0.039387			0.012999
Analytic			0.051682			0.016572
Length=1						
Simulator			0.257218			0.087057
Analytic			0.433715			0.122109

Strategy: Fastest Station First

Station	1	2	3	4	5	6
Rate	600	600	600	300	300	300
Length=½ Speed			4800			7200
Simulator			0.042551			0.013712
Analytic			0.054029			0.017229
Length=1						
Simulator			0.339174			0.092835
Analytic			0.450900			0.127725

Strategy: Random

Station	1	2	3	4	5	6
Rate	300	600	600	300	600	300
Length=½ Speed			4800			7200
Simulator			0.040745			0.014432
Analytic			0.053041			0.016948
Length=1						
Simulator			0.326095			0.089120
Analytic			0.442390			0.125331

Table 4-5. Summary of Constant Box Length Strategies in Preferred Ranking

R=Random S=Slowest First F=Fastest First

Rate=30,90,150			
Speed	2400	4800	7200
Length=1			
Simulator	R,S,F	S,F,R	S,F,R
Analytic	R,S,F	S,R,F	S,R,F
Length=3			
Simulator	S,R,F	S,R,F	S,R,F
Analytic	S,F,R	R,S,F	R,S,F
Rate=300,600			
Speed		4800	7200
Length=½			
Simulator		S,R,F	S,F,R
Analytic		S,R,F	S,R,F
Length=1			
Simulator		S,R,F	S,R,F
Analytic		S,R,F	S,R,F

Table 4-6. Arrangement Delay in Hours for Constant Rate Strategies

Strategy: Shortest First

Station	1	2	3	4	5	6	7	8	9	10
Length	1	1	2	2	2	3	3	3	4	4

	Speed 2400	4800	7200
Rate=30			
Simulator	0.060712	0.009527	0.002948
Analytic	0.080426	0.010821	0.003688
Rate=150			
Simulator	110.074879	1.537758	0.329752
Analytic	10.285448	1.509507	0.406814

Strategy: Longest First

Station	1	2	3	4	5	6	7	8	9	10
Length	4	4	3	3	3	2	2	2	1	1

	Speed 2400	4800	7200
Rate=30			
Simulator	0.079922	0.013163	0.006125
Analytic	0.167165	0.021868	0.007559
Rate=150			
Simulator	19.283618	1.246930	0.330593
Analytic	15.179182	2.925933	0.856376

Strategy: Random

Station	1	2	3	4	5	6	7	8	9	10
Length	2	1	3	4	2	4	1	2	3	3

	Speed 2400	4800	7200
Rate=30			
Simulator	0.067351	0.011630	0.003699
Analytic	0.118935	0.016269	0.005655
Rate=150			
Simulator	64.064156	1.250038	0.307034
Analytic	12.117878	2.053905	0.588017

Table 4-7. Arrangement Delay in Hours for Constant Rate Strategies

Strategy: Shortest First

Station	1	2	3	4	5	6
Length	½	½	½	1	1	1

	Speed 4800	Speed 7200
Rate=300		
Simulator	0.039960	0.013253
Analytic	0.047982	0.015257
Rate=600		
Simulator	0.344046	0.094967
Analytic	0.398627	0.113065

Strategy: Longest First

Station	1	2	3	4	5	6
Length	1	1	1	½	½	½

	Speed 4800	Speed 7200
Rate=300		
Simulator	0.052267	0.020546
Analytic	0.083568	0.026612
Rate=600		
Simulator	0.360787	0.109217
Analytic	0.680261	0.197275

Strategy: Random

Station	1	2	3	4	5	6
Length	½	1	1	½	1	½

	Speed 4800	Speed 7200
Rate=300		
Simulator	0.048011	0.015436
Analytic	0.069781	0.022230
Rate=600		
Simulator	0.348898	0.101927
Analytic	0.571369	0.164590

Table 4-8. Summary of Constant Production Rate Strategies in Preferred Ranking

R=Random	S=Shortest First		L=Longest First
Length=1,2,3,4			
Speed	2400	4800	7200
Rate=30			
Simulator	S,R,L	S,R,L	S,R,L
Analytic	S,R,L	S,R,L	S,R,L
Rate=150			
Simulator	L,R,S	R,L,S	R,S,L
Analytic	S,R,L	S,R,L	S,R,L
Length=½,1			
Speed		4800	7200
Rate=300			
Simulator		S,R,L	S,R,L
Analytic		S,R,L	S,R,L
Rate=600			
Simulator		S,R,L	S,R,L
Analytic		S,R,L	S,R,L

other group of stations that have rates of 300 and 600. By observing the row having a rate of 150 in Table 4-8, it should be observed that the simulator and·analytic models do not agree in the ranking of strategies at the lower speeds. A possible reason for the difference between models is discussed in the next section. If the row having the rate of 150 is omitted from the first group, then the null hypothesis can be rejected at an α-value of 0.10. Thus, the ordering of the strategies in Table 4-8 appear not to be random and indicates that it will be more desirable to use the policy of the shortest length first instead of the other strategics tested. Also, the ranking of policies by the analytical model is similar to the ranking of policies of the simulator.

Test 3 - Mixture of Rates and Lengths

The results in "arrangement delay" of the investigation of strategies involving a mixture of rates and lengths are given in Tables 4-9 and 4-10 and are summarized in Table 4-11. Again Friedman's multi-sample test was used to investigate the rankings in Table 4-11. The null hypothesis was rejected at an α-value of 0.10 for the first group of stations that have rates of 30, 90, and 150. However, due to the way the second group of stations that have rates of 300 and 600 is listed, it was impossible to use Friedman's test. Thus, the ordering of strategies in Table 4-11 are not random for the first group of stations and indicates that it

Table 4-9. Arrangement Delay in Hours for Mixed Station Strategies

Station	A	B	C	D	E	F	G	H	I	J
Rate	30	30	30	30	90	90	90	90	150	150
Length	1	2	3	4	1	2	3	4	1	2
P	30	60	90	120	90	180	270	360	150	300

Strategy: Order in Increasing P; Smallest Rate First in Ties

Station	1-A	2-B	3-C	4-E	5-D	6-I	7-F	8-G	9-J	10-H

Speed	2400	4800	7200
Simulator	0.761188	0.084066	0.026377
Analytic	1.237631	0.141362	0.042188

Strategy: Order in Increasing P; Largest Rate First in Ties

Station	1-A	2-B	3-E	4-C	5-D	6-I	7-F	8-G	9-J	10-H

Speed	2400	4800	7200
Simulator	0.777579	0.081064	0.029509
Analytic	1.198509	0.137383	0.041080

Strategy: Order in Decreasing P; Smallest Rate First in Ties

Station	1-H	2-J	3-G	4-F	5-I	6-D	7-C	8-E	9-B	10-A

Speed	2400	4800	7200
Simulator	0.799228	0.098709	0.035065
Analytic	1.838391	0.220834	0.065613

Strategy: Order in Decreasing P; Largest Rate First in Ties

Station	1-H	2-J	3-G	4-F	5-I	6-D	7-E	8-C	9-B	10-A

Speed	2400	4800	7200
Simulator	0.817371	0.097215	0.034810
Analytic	1.792263	0.217699	0.064952

Strategy: Random

Station	1-F	2-D	3-E	4-B	5-J	6-H	7-C	8-A	9-G	10-I

Speed	2400	4800	7200
Simulator	0.671794	0.082472	0.028462
Analytic	1.701561	0.189543	0.055527

Strategy: Order in Increasing Rate; Smallest Length First

Station	1-A	2-B	3-C	4-D	5-E	6-F	7-G	8-H	9-I	10-J

Speed	2400	4800	7200
Simulator	0.663432	0.090842	0.032886
Analytic	1.665547	0.193168	0.057355

Strategy: Order in Increasing Rate; Largest Length First

Station	1-D	2-C	3-B	4-A	5-H	6-G	7-F	8-E	9-J	10-I

Speed	2400	4800	7200
Simulator	0.691094	0.097907	0.033183
Analytic	2.074469	0.245135	0.071729

Table 4-10. Arrangement Delay in Hours for Mixed Station Strategies

Station	A	B	C	D	E	F
Rate	300	300	300	600	600	600
Length	½	½	1	½	1	1
P	150	150	300	300	600	600

Strategy: Order in Increasing P; Smallest Rate First in Ties

Station	1-A	2-B	3-C	4-D	5-E	6-F

Speed	4800	7200
Simulator	0.126399	0.040252
Analytic	0.176414	0.052766

Strategy: Order in Increasing P; Largest Rate First in Ties

Station	1-A	2-B	3-D	4-C	5-E	6-F

Speed	4800	7200
Simulator	0.120866	0.036949
Analytic	0.164533	0.048995

Strategy: Order in Decreasing P; Smallest Rate First in ties

Station	1-F	2-E	3-C	4-D	5-B	6-A

Speed	4800	7200
Simulator	0.160843	0.047174
Analytic	0.284627	0.084867

Strategy: Order in Decreasing P; Largest Rate First in Ties

Station	1-F	2-E	3-D	4-C	5-B	6-A

Speed	4800	7200
Simulator	0.155743	0.049673
Analytic	0.273247	0.081631

Strategy: Random

Station	1-D	2-F	3-C	4-A	5-E	6-B

Speed	4800	7200
Simulator	0.135506	0.047768
Analytic	0.223552	0.066436

Table 4-11. Summary of Mixed Station Strategies in Preferred Ranking

A= Increasing P with Smallest Rate First in Ties
B= Increasing P with Largest Rate First in Ties
C= Decreasing P with Smallest Rate First in Ties
D= Decreasing P with Largest Rate First in Ties
E= Random
F= Increasing Rate with Smallest Length First
G= Increasing Rate with Largest Length First

Rate=30,90,150	Length=1,2,3,4						
Speed=2400							
Simulator	F	E	G	A	B	C	D
Analytic	B	A	F	E	D	C	G
Speed=4800							
Simulator	B	E	A	F	D	G	C
Analytic	B	A	E	F	D	C	G
Speed=7200							
Simulator	A	E	B	F	G	D	C
Analytic	B	A	E	F	D	C	G
Rate=300,600	Length=½,1						
Speed=4800							
Simulator			B	A	E	D	C
Analytic			B	A	E	D	C
Speed=7200							
Simulator			B	A	C	E	D
Analytic			B	A	E	D	C

is more desirable to use the policy of placing stations in terms of increasing "P" than to use the other strategies tested. As in the previous tests, the ranking of policies by the analytical model is similar to the ranking of policies by the simulator. Before ending the discussion on arrangement strategies, a comment on conveyor utilization is in order. The term conveyor utilization refers to how much conveyor space is expected to be occupied by boxes when no delays occur in the system. Conveyor utilization can be computed by summing the products of the production rate and box length for each station and by dividing the computed sum by the conveyor speed. The values for conveyor utilization for Test 1, Test 2, and Test 3 are given in Table 4-12. It should be noted that the ordering differences between the simulator and analytic models for Test 2 and Test 3 are greatest when the values for conveyor utilization are high. Thus, the ranking of arrangement strategies found may be different for high values for conveyor utilization. In addition, note that the value of 1.56 in Table 4-12 is infeasible, since more space is needed by the system than can be provided by the conveyor speed of 2400 feet per hour.

Table 4-12. Conveyor Utilization for the Three Tests

Test 1			
Speed	2400	4800	7200
Length=1	0.38	0.19	0.13
3	0.75	0.38	0.25
½		0.28	0.19
1		0.56	0.38
Test 2			
Speed	2400	4800	7200
Rate=30	0.31	0.16	0.10
150	1.56	0.78	0.52
300		0.28	0.19
600		0.56	0.38
Test 3			
Speed	2400	4800	7200
Range of Rates			
30,90,150	0.69	0.34	0.23
300,600		0.44	0.29

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

The problem of arranging work stations in series along a continuous conveyor has been investigated in this research. An analytic model describing the expected delay for each station was derived. Also, the simulation model was developed to find the delay for each station. Also, both of these models used station production rates which are Poisson. Furthermore, the stations were not restricted to having identical production rates and box lengths. Both of these models were used in an investigation of a number of station arrangement strategies. Using Friedman's multi-sample test, the rank order of the strategies in terms of lowest arrangement delay was statistically tested. The general findings of the investigation can be summarized as follows:

1. For a constant box length, the strategy of placing the slowest station first was found to outrank the other strategies used;

2. For a constant production rate, the strategy of placing the station with the shortest length first was found to outrank the other strategies used;

3. For a situation in which the production rates and box lengths vary from station to station, the strategy of placing the station with the smallest product of rate and

length first was found to outrank the other strategies used.

The arrangement rules seem to work best when the conveyor utilization is low. However, their performance as compared to other strategies may not be desirable when the conveyor utilization is high or roughly over fifty percent.

In addition, there are many other topics which could be researched in continuous conveyor systems. Instead of observing only the loading area of a continuous conveyor, the unloading area or both loading and unloading areas could be investigated. The development of a more precise analytical model is an additional topic which could be pursued. To undertake such a development would possibly require that individual types of boxes leaving the stations to be explicitly maintained. While computational problems may occur after a number of stations, it may be feasible to determine where the distribution of the conveyor load will approach the normal distribution.

Another topic is the investigation of conveyor systems which use other distributions for describing the interarrival times for each station. This would require that the simulation model be changed to accommodate each distribution used. If the normal distribution is to be studied, then the only major changes necessary would be the inclusion of reading in each station's standard deviation and the use of a normal interarrival time subroutine. Also, if an analytical model is to be developed, it is suggested that tests should be first conducted on the simulator to determine the nature of the distribution of the conveyor load from station to station.

The study of the inclusion of non-delay costs and non-equal delay costs is another topic. The use of arrangement delay as the decision criteria does not readily allow for the use of such costs. However, non-delay costs and non-equal delay costs are also of great importance in industry. In finding design rules to reduce overall costs, the simulator could be used in obtaining an array of delays for each station. The rules can then be compared to one another by using the array to find values for each station's delay costs.

Also, better procedures for finding simulator estimates and for obtaining a larger number of replications could be found. The simulator could be improved by more efficient coding to reduce the amount of computer time necessary for a run. Also, it would be advisable to alter the program by changing the procedure for obtaining arrival times. The arrivals should be first obtained in a sufficient number in a subroutine before placement onto the conveyor. This change would be generally benefical if different distributions were to be used.

Likewise, the investigation of the stations along a continuous conveyor could be expanded by varying the length of the entry range or by using different sets of stations. The reason for using an entry range of two box lengths was to simplify the modeling required in the analytic model. The study of changing the entry range to fit other conveyor loading environments may provide additional information. The sets of stations investigated in Chapter IV could be amended by a study of different systems in which new values for the number of station, box lengths, production rates and speeds are to be given. Such a study should broaden the scope of the results presented in the previous chapter.

Finally, the increases in conveyor speed may cause the differences found between the strategies to be practically negligible. Additional research could be used to find the point at which the differences become insignificant. The simulator with a few minor adjustments would probably be sufficient in finding values for a limited number of stations.

APPENDIX

```
C
C
C
C
C
C        APPENDIX   (A-1)
C
C
C
C SIMULATION OF A CONVEYOR SYSTEM WITH EXPONENTIAL
C        INTERARRIVAL TIMES
C                 AT EACH STATION
C
C
C INITIALIZE:
C       THIS SECTION SPECIFIES THE START OF THE SIMULATION.
C       VALUES FOR THE MEAN NON-DELAYED OUTPUT RATE AND FOR
C        THE BOX
C       LENGTH AT EACH STATION MUST BE GIVEN. CONVEYOR SPEED
C        AND THE
C       TOTAL TIME OF STUDY MUST ALSO BE GIVEN.
C
      PROGRAM JCHSIM(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION JSEED(10)
      DIMENSION RMEAN(10)
      DIMENSION BOSS(10)
      DIMENSION DUMB(5,10)
      DIMENSION FLAGER(10)
      DIMENSION AMEAN(10),AVTIME(10),BOX(10)
      DIMENSION NUMON(10),ICOUNT(10)
      DIMENSION IENSTA(10000)
      DIMENSION ENTRY(10000)
      DIMENSION SELF(10),DELAY(10),AVGDEL(10)
C
      JHNUM=10000
      JHSTA=10
      JHDSM=5
      HDSM=JHDSM
   50 READ(5,100)NUMSTA,SPEED,HOUR
  100 FORMAT(I3,F10.2,F10.2)
      DO 130 I=1,NUMSTA
      READ(5,125)AMEAN(I),BOX(I)
  125 FORMAT(2F8.2)
      DO 126 JOECH=1,JHDSM
      DUMB(JOECH,I)=0.0
  126 CONTINUE
      READ(5,127)JSEED(I)
  127 FORMAT(I8)
  130 CONTINUE
      WRITE(6,131)NUMSTA,SPEED
```

```
131 FORMAT("1",1X,"NUMBER OF STATIONS",I3," CONVEYOR
   *   SPEED",F10.2)
    DO 135 LEDI=1,JHNUM
    ENTRY(LEDI)=99999.99
135 CONTINUE
    DO 150 I=1,NUMSTA
    WRITE(6,136)AMEAN(I),BOX(I),I
136 FORMAT("0",3X,"MEAN RATE",F8.2,"  BOX LENGTH",F8.2,"
   *    STA",I3)
    AVTIME(I)=1/AMEAN(I)
    DELAY(I)=0.0
    FLAGER(I)=0.0
    NUMON(I)=0
    ICOUNT(I)=0
    SELF(I)=0.0
    AVGDEL(I)=0.0
150 CONTINUE
    N1BXS=HOUR/AVTIME(1)+1
    WRITE(6,151)N1BXS
151 FORMAT("-",5X,"THERE WILL BE",I8," 1 TYPE BOXES")
    JHONE=N1BXS
    KOMEIN=1
    NEXT=1
    JUMPIN=2
    START=0.0
    CUMTIM=START
C
C
C GENERATE ARRIVALS.
C      THIS PORTION OF THE SIMULATION WILL CREATE THOSE
C       ARRIVALS
C      TIMES NEEDED TO DETERMINE THE DELAY OCCURRING AT
C       STATIONS.
C      THIS IS A FORWARD PASS PROCEDURE IN WHICH A NUMBER
C      ENTRIES FROM STATION ONE ARE CREATED. AFTER THIS HAS
C       BEEN
C      PERFORMED, ENTRIES FROM THE NEXT STATIONS ARE
C       ENTERED.
C
C GENERATE NEXT ARRIVAL FOR STATION 1
C
    ISEED=JSEED(1)
    IENSTA(1)=1
    ENTRY(1)=0.0-BOX(1)/(2*SPEED)
    END=ENTRY(1)+BOX(1)/SPEED
    NUMON(1)=1
    ZAPOUT=0.0
    DO 310 IJ=2,N1BXS
    CALL GGEXP(ISEED,AVTIME(1),NEXT,TIME)
    ZAPOUT=ZAPOUT+TIME
```

```
      PLACE=CUMTIM+TIME
      ARRIVE=CUMTIM+TIME
      HALF=BOX(1)/(2*SPEED)
      TOTAL=END+HALF
      IF(ARRIVE.GT.TOTAL)GO TO 301
      IF(ARRIVE.GT.END)GO TO 275
      SELF(1)=END-ARRIVE
      TIME=END-CUMTIM
  275 PLACE=TOTAL
      DELTA=TIML
  301 CUMTIM=CUMTIM+TIME
      ENTRY(IJ)=PLACE-BOX(1)/(2*SPEED)
      END=ENTRY(IJ)+BOX(1)/SPEED
      IENSTA(IJ)=1
      NUMON(1)=NUMON(1)+1
  310 CONTINUE
      TEST=ENTRY(N1BXS)
      WRITE(6,315)TEST
  315 FORMAT("0",5X,"TEST TIME",F15.6,"     HRS.")
C
C GENERATE OTHER ARRIVAL TIMES
C
      DO 800 I=2,NUMSTA
      ISEED=JSEED(I)
      ZAPOUT=0.0
      NXENT=JUMPIN
      SBLOCK=0.0
      CUMTIM=START
      WRITE(6,350)CUMTIM,NXENT
  350 FORMAT("0",5X,"CUMTIM",F15.6,"    NXENT",I6)
  360 CALL GGEXP(ISEED,AVTIME(I),NEXT,TIME)
      ZAPOUT=ZAPOUT+TIME
      ARRIVE=CUMTIM+TIME
      IF(TIME.GT.SBLOCK)GO TO 361
      SELF(I)=SBLOCK-TIME+SELF(I)
      TIME=SBLOCK
      DELTA=SBLOCK-TIME
  361 CUMTIM=CUMTIM+TIME
      WEIDLE=DELAY(I)
      NUMON(I)=NUMON(I)+1
C
C FEASIBILITY CHECKS
C       THIS SECTION CHECKS THE ARRIVAL TIME THAT HAS JUST
C        BEEN
C       CREATED. ALL ALTERATIONS TO THIS TIME WILL BE
C        PERFORMED
C       IN THIS SECTION. IF THE "PYRAMID" STRUCTURE IS
C        INVALIDATED
C       BY AN ARRIVAL - THAT TIME WILL BE DISCARDED AND ONLY
C        THE
```

```
C       ARRIVALS FOR THAT STATION WILL BE CONSIDERED.
C
        ISTRIK=0
  700   IF(CUMTIM.GT.ENTRY(NXENT))GO TO 780
        NENT=NXENT-1
        ISTRIK=ISTRIK+1
        IF(ISTRIK.GT.1)GO TO 704
        STRIKE=ENTRY(NENT)+BOX(IENSTA(NENT))/SPEED
        IF(CUMTIM.GT.STRIKE)GO TO 702
        STRIKE=STRIKE-CUMTIM
        GO TO 704
  702   STRIKE=ENTRY(NXENT)-CUMTIM
  704   CONTINUE
C
C     WHAT SPACE EXISTS BETWEEN THESE BOXES ???
C
        SPACE=ENTRY(NXENT)-ENTRY(NENT)-BOX(IENSTA(NENT))/
        DIST=SPACE*SPEED
C
C     IS IT FEASIBLE FOR ENTRY ??
C
        IF(BOX(I).GT.DIST)GO TO 770
C
C     SHIFT RIGHT ?
C
  705   RIGHT=CUMTIM+BOX(I)/(2*SPEED)
        IF(ENTRY(NXENT).LT.RIGHT)GOTO 730
C
C     SHIFT LEFT ?
C
        PORT=CUMTIM-BOX(IENSTA(NENT))/SPEED-BOX(I)/(2*SPEED)
        IF(ENTRY(NENT).GT.PORT)GOTO 740
C
C       EASY   ENTRY   ]]]]
C
        HOLD=ENTRY(NXENT)
        ENTRY(NXENT)=CUMTIM-BOX(I)/(2*SPEED)
        SBLOCK=BOX(I)/(2*SPEED)
        IKEEP=IENSTA(NXENT)
        IENSTA(NXENT)=I
        GO TO 790
C
C       ADJUST    RIGHT   ]]]
C
  730   HOLD=ENTRY(NXENT)
        ENTRY(NXENT)=HOLD-BOX(I)/SPEED
        SBLOCK=HOLD-CUMTIM
        IKEEP=IENSTA(NXENT)
        IENSTA(NXENT)=I
        GO TO 790
```

```
C
C      SHIFT  LEFT  ]]
C
  740 WAIT=ENTRY(NENT)+BOX(IENSTA(NENT))/SPEED
C
C         DOES THE OPERATOR WAIT  ?
C
      IF(CUMTIM.LT.WAIT)GO TO 750
      HOLD=ENTRY(NXENT)
      SBLOCK=BOX(I)/SPEED-CUMTIM+WAIT
      ENTRY(NXENT)=WAIT
      IKEEP=IENSTA(NXENT)
      IENSTA(NXENT)=I
      GO TO 790
C
C         DELAY CAUSED BY LEFT SHIFT .......
C
  750 WAIT=ENTRY(NENT)-CUMTIM+BOX(IENSTA(NENT))/SPEED
      DELAY(I)=DELAY(I)+WAIT
      CUMTIM=CUMTIM+WAIT
      SBLOCK=BOX(I)/SPEED
      HOLD=ENTRY(NXENT)
      ENTRY(NXENT)=ENTRY(NENT)+BOX(IENSTA(NENT))/SPEED
      IKEEP=IENSTA(NXENT)
      IENSTA(NXENT)=I
      GO TO 790
C
C      DELAY CAUSED BY INFEASIBLE ENTRY
C       NOTE! NEXT FEASIBLE ENTRY WILL BE A LEFT SHIFT ]
C
  770 WAIT=ENTRY(NXENT)-CUMTIM+BOX(IENSTA(NXENT))/SPEED
      DELAY(I)=DELAY(I)+WAIT
      CUMTIM=CUMTIM+WAIT
      GO TO 700
C
C LET'S TRY AGAIN
C
  780 NXENT=NXENT+1
      IF(CUMTIM.GT.TEST)GO TO 783
      IF(ENTRY(NXENT).GT.15000.00)GO TO 783
      IF(NXENT.GT.JHNUM)GO TO 785
      GO TO 700
  783 NUMON(I)=NUMON(I)-1
      CUTOFF=DELAY(I)-WEIDLE
      DELAY(I)=WEIDLE
      WRITE(6,784)CUTOFF,WEIDLE,I
  784 FORMAT("0",5X,"DELAY REMOVED",F15.6," REMAINING",
     *  F15.6," STA",I5)
      GO TO 796
  785 WRITE(6,786)I
```

```
  786 FORMAT("0",20X,"DIMENSION ERROR   STATION",I6)
      STOP
C
C LET'S TIE UP THE LOOSE ENDS   (EOS)  END OF SECTION
C
  790 DO 792 LAST=NXENT,JHONE
      NEW=LAST+1
      RETAIN=ENTRY(NEW)
      IALSO=IENSTA(NEW)
      ENTRY(NEW)=HOLD
      IENSTA(NEW)=IKEEP
      HOLD=RETAIN
      IKEEP=IALSO
  792 CONTINUE
      JHONE=JHONE+1
      IF(WEIDLE.LT.DELAY(I))ICOUNT(I)=ICOUNT(I)+1
      IF(WEIDLE.LT.DELAY(I))GO TO 793
      GO TO 360
  793 BIDE=DELAY(I)-WEIDLE
      GO TO 360
  796 FLAGEP(I)=ENTPY(NXENT)
      JUMPIN=2
      IF(I.LT.NUMSTA)GO TO 800
      FLAG=ENTRY(NXENT)
  800 CONTINUE
C
C
C RECORD SECTION
C       AVERAGE DELAY IS FOUND IN THIS SECTION.
C
      WRITE(6,810)
  810 FORMAT("0",5X,"     ")
      WRITE(6,825)
  825 FORMAT("0",10X,"RESULTS  OF  PROCESS
     *   ............")
C
      ALOT=0.0
      DO 900 ID=1,NUMSTA
      ACOUNT=NUMON(ID)
      WRITE(6,850)ACOUNT,DELAY(ID),ID
  850 FORMAT("0",5X,"FOR",F15.6," ENTRIES  DELAY",F15.6,"
     *  HR STA",I3)
      WRITE(6,860)ICOUNT(ID)
  860 FORMAT("0",6X,"OF THESE ENTRIES ",I6,"   HAVE BEEN
     *  DELAYED")
      DELAY(ID)=DELAY(ID)/ACOUNT
      ANUMBR=ICOUNT(ID)
      ANUMBP=ANUMBR/ACOUNT
      WRITE(6,865)ANUMBR
  865 FORMAT("0",10X,"PERCENT DELAYED",F15.6)
```

```
      WRITE(6,875)DELAY(ID)
  875 FORMAT("0",6X,"THIS WILL GIVE AN AVG DELAY",F15.6)
      DUMB(JOECH,ID)=DELAY(ID)
      WRITE(6,880)SELF(ID),ID
  880 FORMAT("0",15X,"SELF BLOCK DELAY",F15.6,5X,"  STA",I3)
      ALOT=NUMON(ID)+ALOT
  900 CONTINUE
      ILOT=ALOT
      WRITE(6,925)ALOT,ILOT
  925 FORMAT("0",6X,"THERE IS",F15.6," ENTRIES",I9)
C
C
C      ANALYTIC    MODEL
C
      DO 1550 I=1,NUMSTA
      EPLI=(AMEAN(I)*BOX(I)/(2*SPEED))*(-1)
      EPL=AMEAN(I)*BOX(I)/SPEED*(-1)
      EX=EXP(EPLI)
      EY=EXP(EPL)
      ETT1=(BOX(I)/(2*SPEED))*(1-EX)
      EXXX=BOX(I)/(2*SPEED)+1/AMEAN(I)
      EXXY=BOX(I)/SPEED+1/AMEAN(I)
      ETT2=EX*(EX*EXXX-EY*EXXY)
      ETT=ETT1+ETT2
      EPZ=(AMEAN(I)*(BOX(I)/(2*SPEED)+ETT))*(-1)
      EZ=EXP(EPZ)
      ESB=1/AMEAN(I)-(BOX(I)/(2*SPEED)+ETT+1/AMEAN(I))*EZ
      RMEAN(I)=AMEAN(I)/(1+ESB*AMEAN(I))
      WRITE(6,1500)ESB,RMEAN(I),ETT
 1500 FORMAT("0",15X,"ESB",F15.6,"  RMEAN",F15.6,"  ETT",
     *  F15.6)
 1550 CONTINUE
      EXBOX=BOX(1)
      EXMEAN=RMEAN(1)
      DO 2000 I=2,NUMSTA
      EA=(EXMEAN*EXBOX)/SPEED*(-1)
      EB=EXMEAN*BOX(I)/SPEED*(-1)
      E3A=-1.5*EXMEAN*EXBOX/SPEED
      E32AB=EXMEAN*(3*EXBOX+2*BOX(I))/(-2*SPEED)
      EA2=EXMEAN*EXBOX/(-2*SPEED)
      EA=EXP(EA)
      EB=EXP(EB)
      E3A=EXP(E3A)
      E32AB=EXP(E32AB)
      EA2=EXP(EA2)
      PBN=EXBOX*EXMEAN/SPEED
      PS=1-PBN
      POSL2=E3A-E32AB+E3A*(1-EB)*(1-EA)/2
      PL2S=E32AB*(1.5-EA/2)
      PSL2=1-PL2S
```

```
      ESSS=1/EXMEAN-((EXMEAN*BOX(I)+SPEED)/(EXMEAN*SPEED))
     *   *EB
      EOSL2=(1.5*EA2-E3A/2)*EA*ESSS
      ESL2=EOSL2/PSL2
      EOSL2=EOSL2/POSL2
      ESPACE=1/EXMEAN-EXBOX/SPEED
      ETA=PSL2/PL2S
      ETB=(ESL2+EXBOX/SPEED)*ETA
      ETC=PBN*(EXBOX/(2*SPEED)+ETB)
      ETD=EOSL2/2+EXBOX/SPEED+ETB
      ETD=PS*(EOSL2*POSL2)*ETD/(ESPACE+EXBOX/SPEED)
      ETN=ETC+ETD
      OUTPUT=RMEAN(I)/(1+RMEAN(I)*ETN)
      OUTN=OUTPUT*PS*(1-PSL2)
      OUTMN=OUTPUT-OUTN
      OUTM=EXMEAN-OUTMN
      PNBN=OUTN*BOX(I)/SPEED
      PNBMN=OUTMN*(EXBOX+BOX(I))/SPEED
      PNBM=OUTM*EXBOX/SPEED
      SUM=PNBN+PNBMN+PNBM
      PN=PNBN/SUM
      PMN=PNBMN/SUM
      PM=1-PN-PMN
      EXBOX=PM*EXBOX+PMN*(EXBOX+BOX(I))+PN*BOX(I)
      EXMEAN=EXMEAN+OUTN
      WRITE(6,1600)
 1600 FORMAT("0",5X,"          ANALYTIC  MODEL")
      BOSS(I)=ETN
      WRITE(6,1700)I,ETN
 1700 FORMAT("0",10X,"STATION",I6,"  HAS BEEN DELAYED",
     *   F15.6)
      WRITE(6,1800)EXBOX,EXMEAN
 1800 FORMAT("0",15X,"AVERAGE BOX",F15.6,"  AVERAGE MEAN",
     *   F15.6)
      DUMB(JOECH,I)=DELAY(I)
      DIFF=DELAY(I)-ETN
      WRITE(6,1810)EXMEAN,EA,EB
 1810 FORMAT("0",10X,"EXMEAN",F15.6,"  EA",F15.6,"  EB",
     *   F15.6)
      WRITE(6,1820)E3A,E32AB,EA2
 1820 FORMAT("0",10X,"E3A",F15.6,"  E32AB",F15.6,"  EA2",
     *   F15.6)
      WRITE(6,1830)PBN,PS
 1830 FORMAT("0",15X,"PBN",F15.6,"   PS",F15.6)
      WRITE(6,1840)POSL2,PL2S,PSL2
 1840 FORMAT("0",10X,"POSL2",F15.6,"  PL2S",F15.6,"  PSL2",
     *   F15.6)
      WRITE(6,1850)ESL2,EOSL2,ESPACE
 1850 FORMAT("0",10X,"ESL2",F15.6,"  EOSL2",F15.6,"
     *   ESPACE",F15.6)
```

```
      WRITE(6,1860)ETA,ETB
 1860 FORMAT("0",10X,"ETA",F15.6,"       ETB",F15.6)
      WRITE(6,1870)ETC,ETD
 1870 FORMAT("0",20X,"ETC",F15.6,"      ETD",F15.6)
      WRITE(6,1880)OUTPUT,OUTN,OUTMN,OUTM
 1880 FORMAT("0",10X,"OUTPUT",F15.6," N",F15.6," MN",F15.6,
     *  " M",F15.6)
      WRITE(6,1890)PNBN,PNBMN,PNBM
 1890 FORMAT("0",15X,"PNBN",F15.6,"  PNBMN",F15.6,"  PNBM",
     *  F15.6)
      WRITE(6,1892)PN,PMN,PM
 1892 FORMAT("0",15X,"PN",F15.6."  PMN",F15.6."  PM",F15.6)
      WRITE(6,1895)EXBOX
 1895 FORMAT("0",30X,"EXBOX",F15.6)
      WRITE(6,1900)DIFF
 1900 FORMAT("0",30X,"SIMULATOR-ANALYTIC",F15.6)
 2000 CONTINUE
      WRITE(6,2002)
 2002 FORMAT("1",5X,"         ")
      DO 2010 I=1,NUMSTA
      WRITE(6,2005)DELAY(I),I
 2005 FORMAT("0",5X,"DELAY/CYCLE",F15.6,"    FOR STATION",I6)
      WRITE(6,2007)BOSS(I)
 2007 FORMAT(" ",15X,"ETN",F15.6)
 2010 CONTINUE
      XBAR=0.0
      ABAR=0.0
      DO 2025 I=1,NUMSTA
      DELAY(I)=AMEAN(I)*DELAY(I)
      BOSS(I)=AMEAN(I)*BOSS(I)
      XBAR=XBAR+DELAY(I)
      ABAR=ABAR+BOSS(I)
       WRITE(6,2015)DELAY(I),I
 2015 FORMAT("0",5X,"DELAY/HOUR",F15.6,"    FOR STATION",I6)
      WRITE(6,2020)BOSS(I)
 2020 FORMAT(" ",15X," OR",F15.6)
 2025 CONTINUE
      WRITE(6,2030)XBAR,ABAR
 2030 FORMAT("0",10X,"TOTAL DELAY/HOUR",F15.6,20X,"OR",
     *  F15.6)
      STOP
      END
```

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